Chemical Engineering Course Pack

Fall 2021

CHEE 314: Fluid Mechanics

Offered by:

CHEMICAL

ChESS

Chemical Engineering Students' Society

CHEE 314: Fluid Mechanics

Dear U2s,

This course pack contains past midterms, quizzes and final exams to help you study for CHEE 314: Fluid Mechanics. As amazing as it sounds, you should ONLY come to this course pack for additional practice problems. You have a great instructor and an easy-to-read textbook, which should be your first points of reference. Make sure you attend all of the lectures because CHEE 314 is a very math-intensive class and Professor Maric provides you with tons of examples to understand the concepts.

Engineering isn't easy, but you wouldn't be here if it was. Very few people become high achieving students overnight. Work hard and you will get your desired outcome at the end.

Some caveats about this course pack:

- Instructors for CHEE 314 have changed throughout the years so ALL the questions in this course pack are not from Professor Maric. Therefore, please do not come to him with any concerns about the course pack. He has made it very clear that he will not endorse and bear any responsibility for this course pack.
- The ChESS council is not responsible for the correctness of the solutions and they may not be complete. If this is the case, compare with your classmates. Also keep in mind that at times, there may be more than one way to find the correct solution.

Some tips to success:

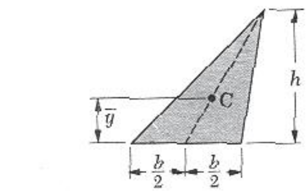
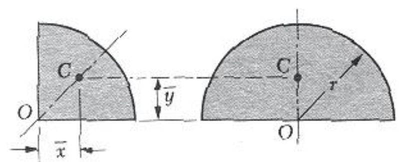
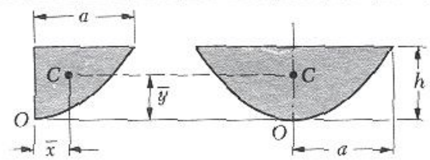
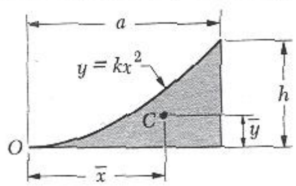
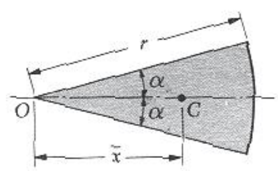
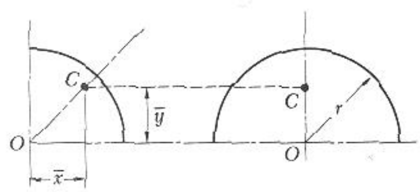
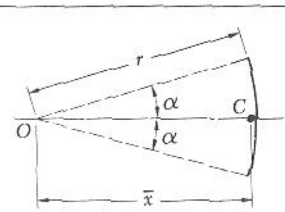
- Do not cram before your exams! Instead, try to keep up with the class and spread out your studying.
- Sit down and do all of the assigned problems (preferably by yourself first then compare with your friends). Make sure you understand every problem.
- For the midterm and quizzes, try to do as many problems as you can from the book exercises. They are great practice.
- Follow the units, sometimes they can give you great hints about problems.
- What's open to atmosphere is atmospheric pressure (if this doesn't make any sense to you now, don't worry)

Keep an eye out for ChESS activities throughout the year such as (possibly) Apartment Crawl, Banquets, Ski Trip, Blues Pub, etc. These events are a great way to take a break and meet students in different years. As always, if you have any questions, please email me at chess.vpacademic@mcgilleus.ca.

Good luck!

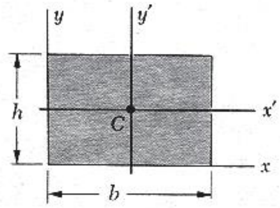
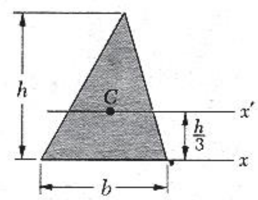
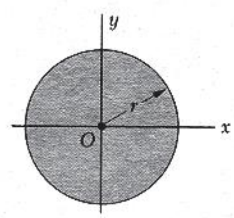
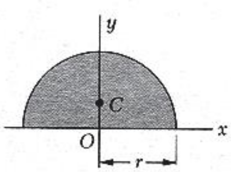
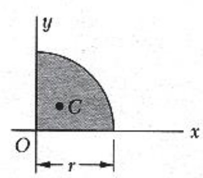
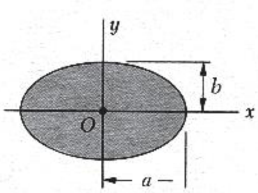
Cheers,
Thinh Bui
ChESS VP Academic 2021-2022

Centroids of Common Shapes of Areas and Lines

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	αr^2
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

Taken from Mechanics of Materials
Beer & Johnson McGraw-Hill 1989

Moments of Inertia of Common Geometric Shapes

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

Taken from Mechanics of Materials
Beer & Johnson McGraw-Hill 1989

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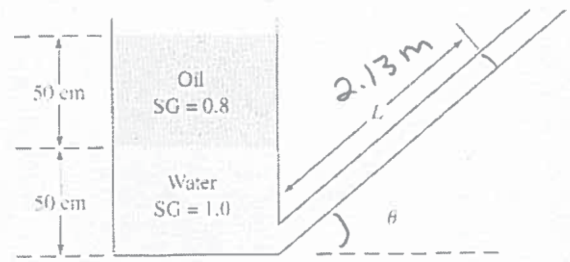
McGill University

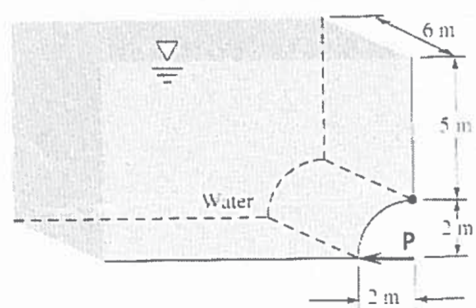
CHEE 314 Fluid Dynamics

Quiz 1 Fall 2013

1. **(5 marks)** In the figure below, both the tank and tube are open to the atmosphere. If L=2.13 m, what is the angle of tilt of the tube (θ)?

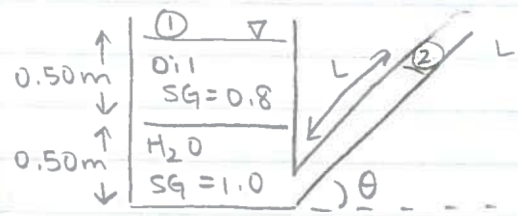


2. **(10 marks)** Find the force P needed to hold close the base of the tank if it is pinned 2 m from the bottom of the tank as shown in the figure. Assume the water is at 20°C. You may neglect the weight of the gate.



Quiz 1 2013

1)



$L = 2.13m$

assume: - static
- incompressible

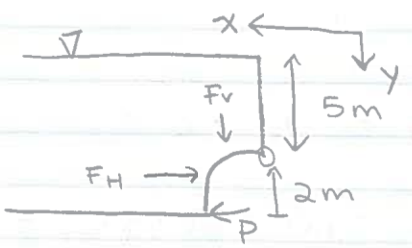
$P_2 = P_1 + 0.8\gamma_{H_2O}(0.50m) + 1.0\gamma_{H_2O}(0.50m) - \gamma_{H_2O} L\sin\theta$

$P_1 = P_2 = 0$

$\sin\theta = \dfrac{0.8\cancel{\gamma}(0.50m) + \cancel{\gamma}(0.50m)}{(2.13m)\cancel{\gamma}}$

$\boxed{\theta = 25^\circ}$

2)



assume: - static
- incompressible
- neglect mass of gate

must break F_R up into horizontal & vertical forces

horizontal

$F_H = \gamma h_c A$

from top of water

$h_c = 5m + 1m = 6m$

$= (9810N/m^3)(6m)(6m)(2m)$

$= 706\ 320N$

$y' = \dfrac{I_{\hat{x}\hat{x}}}{y_c A} + y_c$

for rectangular projection

$I_{\hat{x}\hat{x}} = \frac{1}{12}bh^3$ $\quad y_c = h_c$

$= \dfrac{\frac{1}{12}(6m)(2m)^3}{(6m)(6m)(2m)} + 6m = 6.06m$

vertical split up

Fv_1 acts at x_1

Fv_2 acts at x_2

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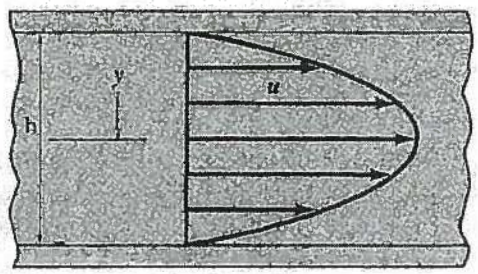
CHEE 314 Fluid Dynamics

Quiz 1 | Fall 2012

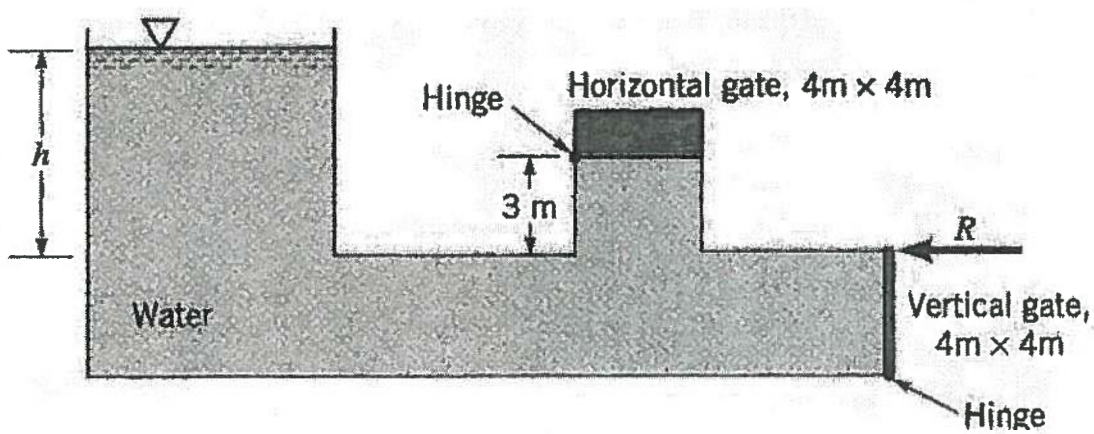
1. **(5 marks)** The velocity distribution for laminar flow between parallel plates is given by:

$$\frac{u}{u_{max}} = 1 - \left(\frac{2y}{h}\right)^2$$



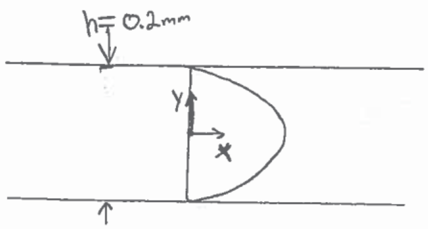
where h is the distance between the plates and the origin is is the middle of the two plates. Consider a flow of water at 15°C with a u_{max}=0.15 m/s and h=0.2 mm. Calculate the shear stress on the upper plate created by the water. Draw the fluid shear stress profile throughout the channel.

2. **(10 marks)** Two square gates close two openings in a conduit connected to an open tank of water shown in the figure. When the water depth, h, reaches 5 m it is desired that both gates open at the same time. Determine the weight of the homogeneous horizontal gate and the horizontal force R acting on the vertical gate that is required to keep the gates closed until the depth h=5 m. The weight of the vertical gate can be neglected and both gates are hinged at one end as shown in the figure.



Quiz 1. Fall 2012 Solutions

1.



Assumptions:
- Newtonian fluid
- No slip at wall
- Infinit plate
- Steady

Given:
$U_{max} = 0.15 \frac{m}{s}$
$h = 0.2mm$
$T = 15°C$
Fluid: water

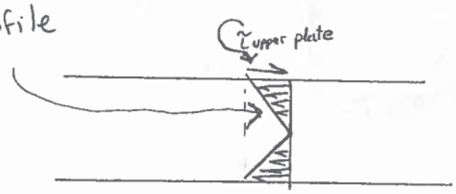
$$\frac{u}{U_{max}} = 1 - \left(\frac{2y}{h}\right)^2 \rightsquigarrow u = 0.15\,m/s\left(1 - \frac{4y^2}{(0.2\times10^{-3}m)^2}\right)$$

$$\tau_{upper} = \mu \frac{du}{dy} \qquad \mu_{H_2O\,(15°C)} = 1.14\times10^{-3}\frac{N\cdot s}{m^2} \qquad \frac{du}{dy} = \frac{-8y(0.15\,m/s)}{(0.2\times10^{-3}m)^2}$$

$$\tau_{upper} = 1.14\times10^{-3}\frac{N\cdot s}{m^2} \times \frac{(-8)(0.1\times10^{-3}m)\times 0.15\,m/s}{(0.2\times10^{-3}m)^2} = -3.42\,Pa$$

Note: τ_{upper} is the shear stress of the fluid, the shear stress on the upper plate created by the fluid is $\rightsquigarrow \tau_{upper\,plate} = -\tau_{upper} = \underline{3.42\,Pa}$

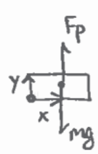
Shear profile of fluid



2. First do horizontal gate

Assume:
- Incompressible
- Static
- T = 10°C
- Homogenous density of gates.

FBD:



mg & F_p act at centroid.

$\Sigma M_o = F_p \cancel{x_c} - mg\,\cancel{x_c} = 0$ at opening point

Pressure at horizontal gate is equal to 5m − 3m = 2m of H_2O

Consider Pgage $\Delta P = g\rho_w h_w = g\times(1000\frac{kg}{m^3}\times 2m)$
$= 19620\,Pa$

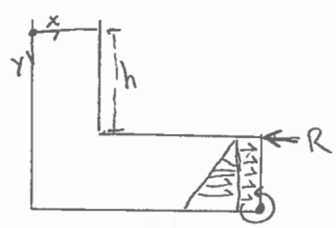
$F_p = 19620\,Pa \times 4m \times 4m = 313.9\,kN$

$F_p - mg = 0$

$F_p = mg = 314\,kN$

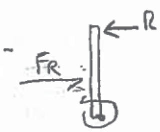
gate should weight $\underline{314\,kN}$

2. Second do Vertical Gate



Assume:
- Incompressible
- Static
- Mass of gate = 0
- T = 10°C

FBD -



$F_R = \gamma h_c A = \gamma(h+2)A$
$= 9810\times 7\times 16 = 1099\,kN$

$$y' = \frac{I_{xx}}{A y_c} + y_c = \frac{\frac{1}{12}bh^3}{A(h+2)} + (h+2) = 7.19m$$

$\Sigma M_F = 0$

$R\times 4m - F_R(9-y') = 0$

$\underline{R = 4.97\times10^5\,N}$

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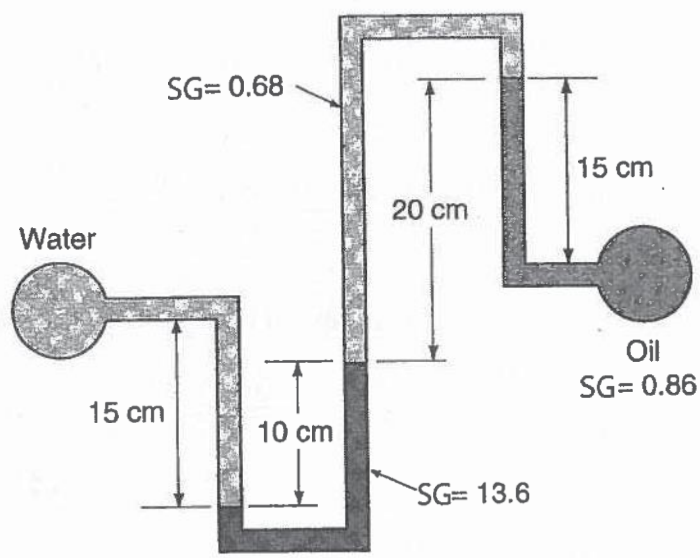
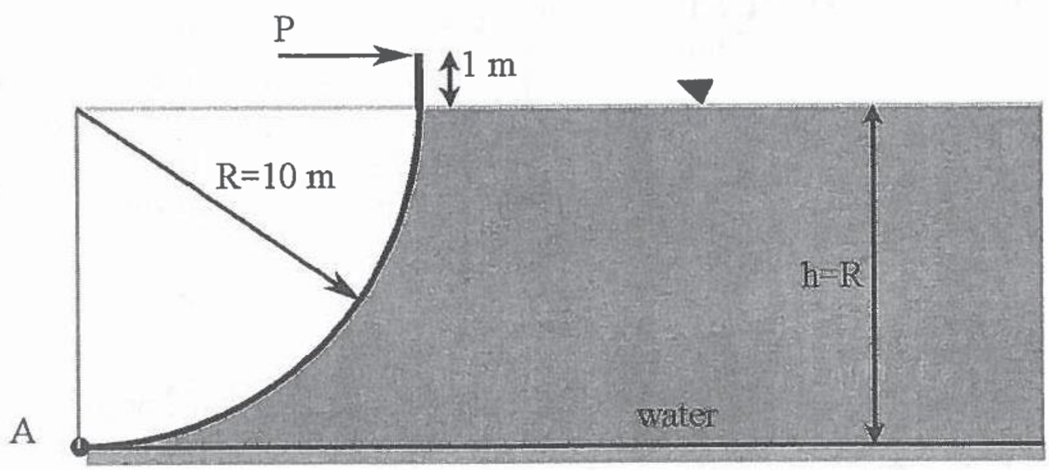
CHEE 314 Fluid Dynamics

Quiz 1 Fall 2011

1. **(5 marks)** Determine the pressure difference between the water pipe and oil pipe shown in the figure.

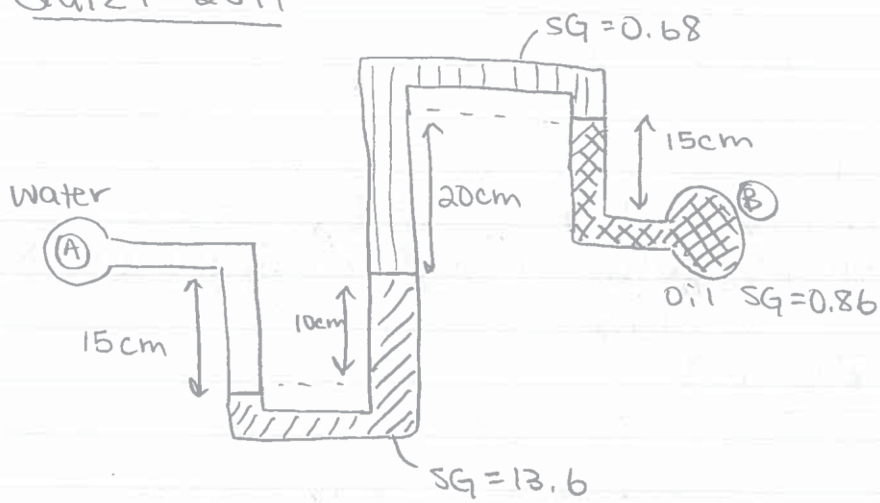


2. **(10 marks)** A reservoir of water is held back by a circular arc gate. The reservoir, filled with water at 10°C, has a depth of 10m and width of 10m. Calculate the force P needed to hold the gate in place if the gate is pinned at point A. You can assume the gate is massless.



Quiz 1 2011

1)



assume:
- incompressible
- static

$$P_A + \gamma_{H_2O}(0.15m) - 13.6\gamma_{H_2O}(0.10m) - 0.68\gamma_{H_2O}(0.20m) + 0.86\gamma_{H_2O}(0.15m) = P_B$$

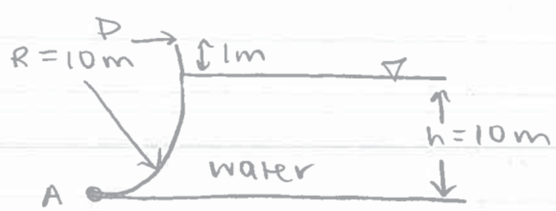
$$P_B - P_A = \gamma_{H_2O}(0.15 - 13.6(0.10) - 0.68(0.20) + 0.86(0.15))$$

$$= (998\ kg/m^3)(9.81\ N/kg)(-1.217)$$

$$= -11\,915\ Pa$$

$$= \boxed{-11.9\ kPa}$$

2)



width 10m

assume:
- static
- incompressible
- mass gate = 0

$$F_H = \gamma_{H_2O} h_c A$$

$$= (998 kg/m^3)(9.81 N/kg)(5m)(10m)(10m)$$

$$= 4\,895\,190\ N$$

$$y' = \frac{I_{\hat{x}\hat{x}}}{y_c A} + y_c$$

$$= \frac{1/12(10m)(10m)^3}{(5m)(10m)^2} + 5m$$

$$= 6.67m$$

$$F_V = \gamma_{H_2O} \forall$$

$$= (998 kg/m^3)(9.81 N/kg)(\pi/4)(10m)^2(10m)$$

$$= 7.69 \times 10^6\ N$$

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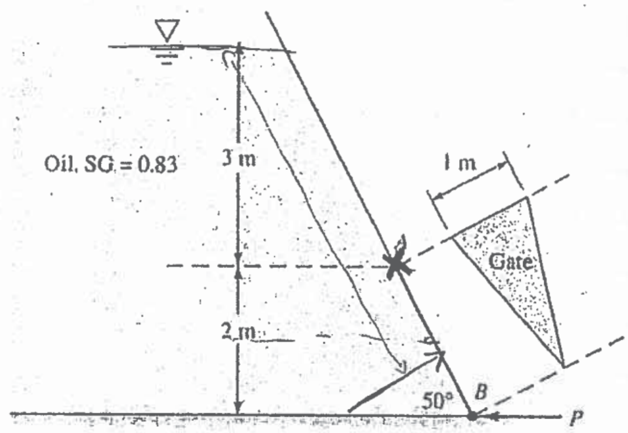
CHEE 314 Fluid Dynamics

Quiz 1 — Fall 2010

1. **(10 marks)** A triangular gate is hinged at point A and weighs 1500 N. What horizontal force P needs to be applied at B to hold the gate closed?

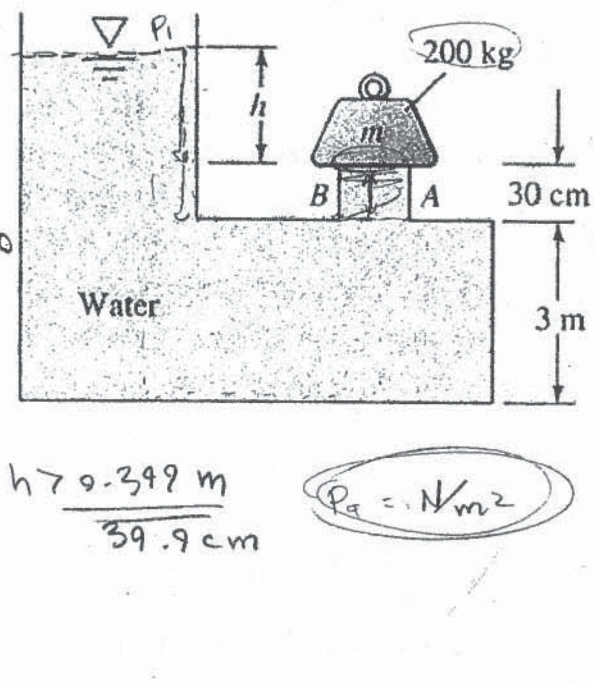


$P = \frac{N}{m^2}$ $\quad \frac{kg\,m}{s^2}$

$= \frac{kg}{m s^2}$

22 kN

2. **(5 marks)** A 200 kg weight is placed on a circular opening (AB) which has a diameter of 80 cm. Assume the water is at 20°C. At what height h will the weight be dislodged?



$P_1 + \rho g h_1 - \rho g(30\,cm) - P_2 = 0$

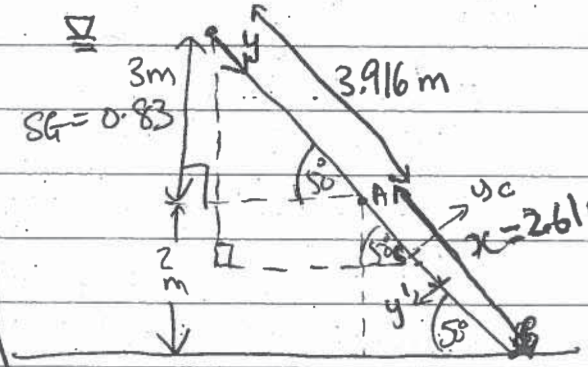
$P_1 + \rho g(h_1 - 30\,cm) - P_2 = 0$

$h > 0.399\ m$

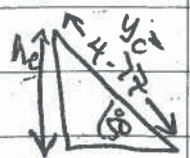
$\underline{39.9\ cm}$

$P_a = N/m^2$

Question 1

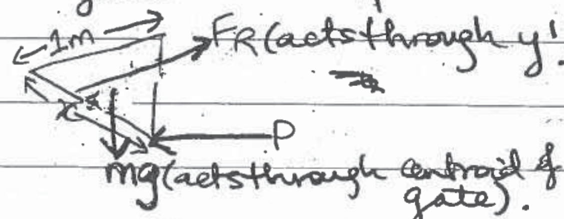


Assumptions
- Patm acts on surface of oil and on outside of gate
- Static fluid
- water @ 20°C
- Incompressible fluid (ρ = constant).
- gate has uniform density



$\sin 50 = \frac{h_c}{y_c}$

$\therefore h_c = y_c \sin 50$

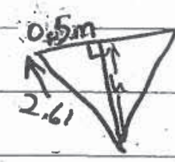


FBD on gate

weight mg acts through the centroid of the triangular gate

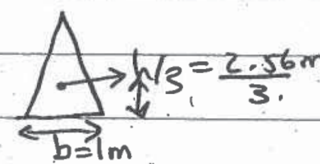
$$\sin 50° = \frac{2}{x} \quad \therefore \quad x = \frac{2}{\sin 50} = 2.61\,m$$

Thus perpendicular height of triangular gate, h



$$h = \sqrt{2.61^2 - 0.5^2} = 2.56\,m.$$

Centroid of triangular gate (right side up) $= \frac{2.56}{3} = 0.85\bar{3}\,m$



$$\therefore \quad y_c = 3.916 + 0.85\bar{3} = 4.77\,m$$

$$y' = y_c + \frac{I_{xx}}{Ay_c} \qquad I_{xx} \text{ for triangle} = \frac{1}{36}bh^3$$

$$\therefore y' = 4.77 + \frac{\frac{1}{36}\cancel{b}\cancel{h^3}h^2}{\frac{1}{2}\cancel{bh}\,y_c} = 4.77 + \frac{2}{36}\frac{h^2}{4.77} = 4.85\,m$$

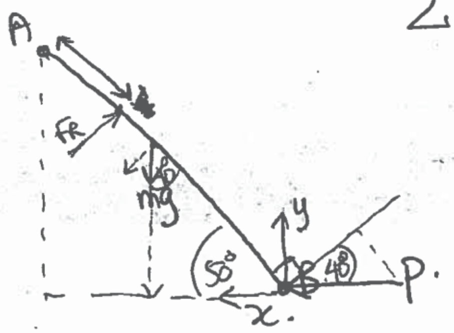
$h_c = y_c \sin 50 = 3.65\,m$

$$F_R = \gamma A_{gate} h_c = SG_{oil}\,\rho_{H_2O}\,g\,A_{gate}\,h_c = (0.83)(998)(9.81)(\tfrac{1}{2}(2.56)(1))(3.6$$

$$F_R = 37.96\,kN.$$

Taking moments about point A.

$\sum M_{o@A} = 0$ for equilibrium



$$0 = F_R(4.85-3.92) - F_{weight}\sin(40^\circ)(0.853) - P\sin 50(\ldots$$

$$\therefore 0 = (37.96\times10^3)(0.93) - (1500)(\sin 40)(0.853) - P(\sin 50)(2.61)$$

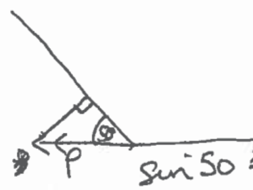
$\sin 40^\circ = \frac{F_{Hweight}}{F_{weight}}$

$\therefore F_{H weight} = F_{weight}\sin 40$

$$0 = 35303 - 822 - 1.999P$$

$$\therefore 1.999P = 35303 - 822$$

$$\boxed{\therefore P = 17.25\,\text{kN}}$$



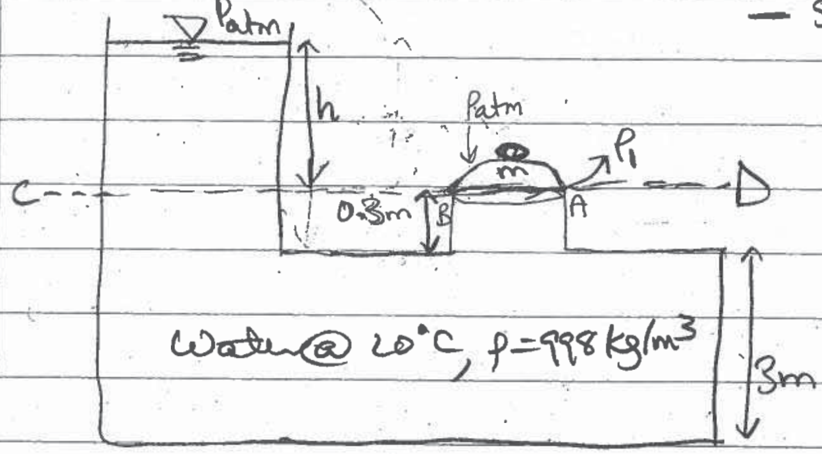
$\sin 50 = \frac{P_H}{P}$

$\therefore P_H = P\sin 50$

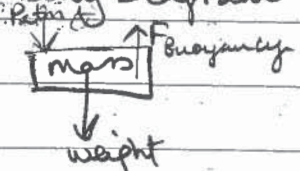
10

Question 2

Assume — incompressible fluid
— Static fluid



Free Body Diagram



From the FBD, we see P_{atm} cancels out, thus we may work in gage pressures

At liquid level CD

$$\cancel{P_{atm}} + \rho g h + 0.3\rho g - 0.3\rho g = P_1 + \cancel{P_{atm}}$$

$$\therefore \quad P_1 = \rho g h$$

To dislodge weight, $F_B = P_1(\text{Area of opening for mass})$ (cross sectional) $\geq F_{weight}$

$$\Rightarrow \quad \rho \cancel{g} h \pi r^2 > m \cancel{g}$$

$$\therefore \ h > \frac{m}{\rho \pi r^2} = \frac{200}{998 \times \pi \times \left(\frac{80}{2} \times 10^{-2}\right)^2}$$

$$h > \frac{200}{501.65} = 0.399\,m$$

The weight will thus be dislodged @ $h > 0.399\ m$

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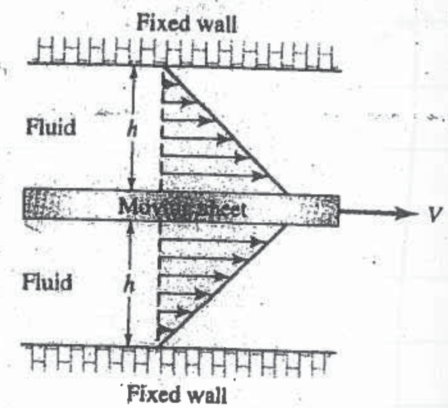
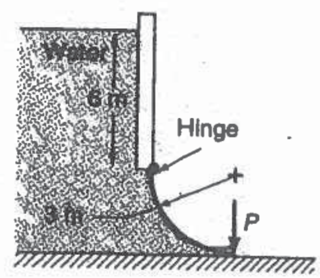
CHEE 314 Fluid Dynamics

Quiz 1 Fall 2007

1. **(5 marks)** A thin sheet of steel is pulled through an oil bath. The oil has a viscosity of 0.02 kg/(m·s). If h=4m and the total oil-steel contact area is 20 m^2, calculate the force necessary to move the steel at a velocity of 5 m/s. You can assume the velocity profile is linear.



2. **(5 marks)** What force P is needed to hold the 4 m wide gate closed? Assume the water temperature is 20°C.



Quiz Solutions

① Assume

- Newtonian
- linear velocity profile
- steady
- no slip

$$\tau = \mu \cdot \frac{du}{dy} = \mu\left(\frac{0-v}{h-0}\right) = -\frac{\mu v}{h}$$

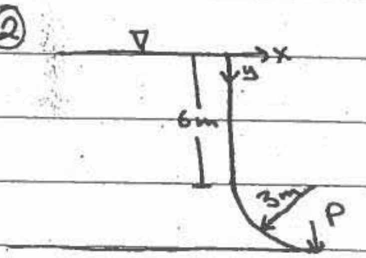
shear of fluid on the plate

↳ has to be overcome by equal & opposite force

∴ the force required $F = \tau A$

$$F = \left(\frac{\mu v}{h}\right)(20\ m^2) = 0.5\ N$$

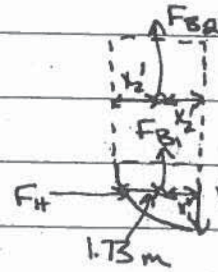
②



Assume

- static
- incompressible

FBD



$$F_H = \gamma A_{proj.} h_{c proj.}$$

$$= 9790(3\times4)(7.5)$$

$$= 8.81\times10^5\ N$$

$$y' = \frac{I_{\hat{x}\hat{x}}}{A y_{c proj.}} + y_{c proj.}$$

$$= \frac{\frac{1}{12}(4)(3)^3}{4\times3(7.5)} + (7.5) = 7.6 m$$

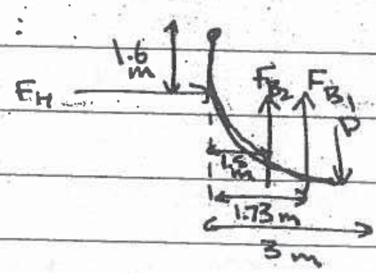
$F_{B_1} = \gamma V_1$
$= 9790\left(\frac{\pi}{4}(3)^2 \times 4\right)$
$= 2.77 \times 10^5$ N

$x_1' = \frac{4r}{3\pi}$

$F_{B_2} = \gamma V_2$
$= 9790(6 \times 3 \times 4)$
$= 7.05 \times 10^5$ N

$x_2' = 1.5$ m

Moments:



$\sum M_o = 0$

$(8.81 \times 10^5)(1.6) + (7.05 \times 10^5)(1.5) + (2.77 \times 10^5)(1.73) - P(3) = 0$

$P = 9.82 \times 10^5$ N

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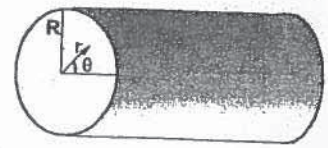
CHEE 314 Fluid Dynamics

Quiz 1 | Fall 2006

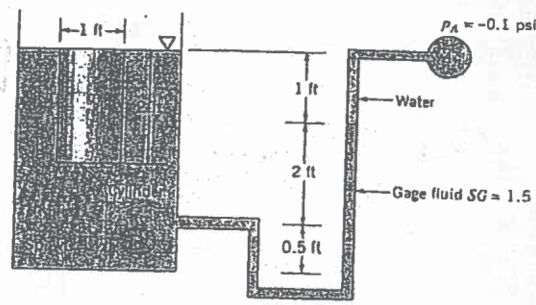
1. **(5 marks)** The velocity distribution for laminar flow in a pipe is given by



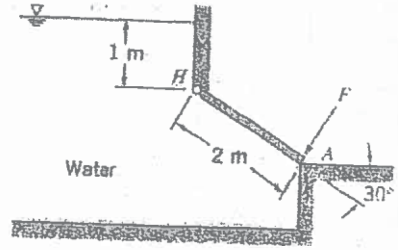
$$u(r) = \frac{R^2}{4\mu}\frac{dP}{dx}\left[1-\left(\frac{r}{R}\right)^2\right]$$

Where R is the radius of the pipe, dP/dx and μ are constant. What is the shear stress on the wall of the pipe? Sketch the variation of shear stress across the pipe.

2. **(5 marks)** A 1-ft diameter, 2-ft long cylinder floats in an open tank containing a liquid having a specific weight of γ. A U-tube manometer is connected to the tank as shown. The pressure in pipe A is below atmospheric pressure (-0.1 psi). Using the various fluid levels determine a) the specific weight of the liquid b) the weight of the cylinder.



3. **(5 marks)** The gate shown is hinged at H. The gate is 2 m wide and 2 m long. Calculate the force F needed at point A to hold the gate closed.



Q1 $$u(r) = \frac{R^2}{4\mu}\frac{dP}{dx}\left(1 - \left(\frac{r}{R}\right)^2\right)$$

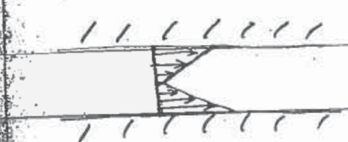
Assume

- Steady
- Newtonian

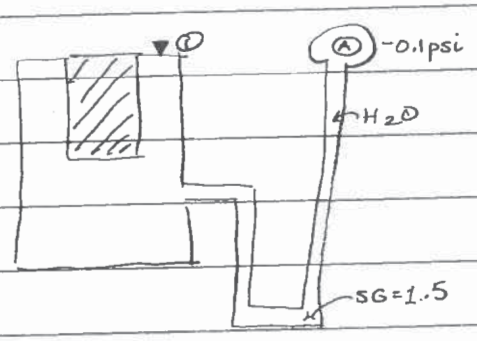
$$\tau = \mu \frac{du}{dr}$$

$$= \frac{\mu R^2}{4\mu}\frac{dP}{dx}\left(\frac{-2r}{R^2}\right)$$

$$\tau @ r=R \qquad = \frac{-R}{2}\frac{dP}{dx}$$



Q2



Assume

- Static
- Incompressible

a) $\cancel{P_1}^{\,0} + 3.5\gamma - 2.5(1.5\gamma_{H_2O,4°C}) - 1\gamma_{H_2O} = -0.1 \frac{lb_f}{in^2}\left(\frac{144 in^2}{ft^2}\right)$

$$\gamma = 80.6 \frac{lb_f}{ft^3} = 1267.1 \frac{Kg}{m^3}$$

b)

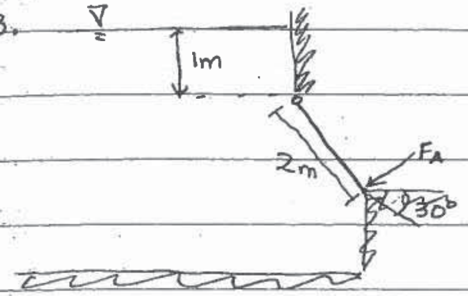
FBD

$$\sum F_y = 0$$

$$\gamma \forall = mg$$

$$= 127 \; lb_f$$

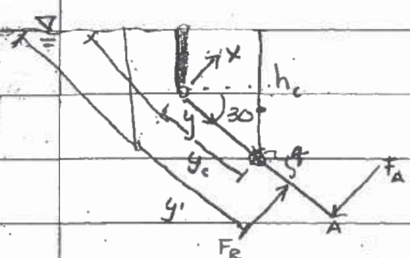
3.



Assume

- static
- Incompressible
- $m_{gate} = 0$
- Neglect atmospheric P
- $T = 20°C$

FBD

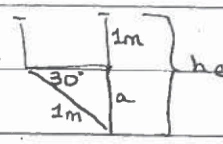


$F_R = \gamma A h_c$

$= \rho g (2m)(2m)(1+\sin 30° m)$

$= (998\ kg/m^3)(9.81\ m/s^2)(4m^2)(1+\sin 30° m)$

$= 58742.28\ N$

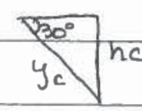


$\sin(30°) = a$

$h_c = \sin(30°)+1$

$y' = \frac{I_{xx}}{A y_c} + y_c$

$y' = \frac{\frac{1}{12}(b)(h)^{3\,2}}{(b)(h)(1+\frac{1}{\sin 30})} + 1 + \frac{1}{\sin 30}$



$= \frac{(2)^2}{(1+\frac{1}{\sin 30})} + 1 + \frac{1}{\sin 30}$

$\sin 30° = \frac{h_c}{y_c}$

$y_c = \frac{h_c}{\sin 30°}$

$y' = \frac{4}{3} + 1 + 2 = 3.111\ m$

$y_c = \frac{\sin 30 + 1}{\sin 30}$

$y_c = 1 + \frac{1}{\sin 30°}$

$= 3$

$\Sigma M_{hinge} = 0$

$y' - y^c = a$

$F_R(1.111) = F_A(2)$

$3.111 - 3 = a \qquad a = .111 + 1$

$38\ KN = F_A$

F_A to hold the gate closed $= 38\ KN$

4/5

Department of Chemical Engineering
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CHEE 314 Fluid Dynamics

Midterm — Fall 2013

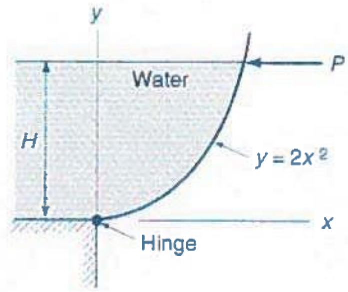
1. (8 marks) The velocity components in a cylindrical coordinate system are given by:

$$u_r = \left(10 - \frac{40}{r^2}\right) cos\theta\ , \qquad u_\theta = -\left(10 + \frac{40}{r^2}\right) sin\theta, \qquad u_z = 0$$

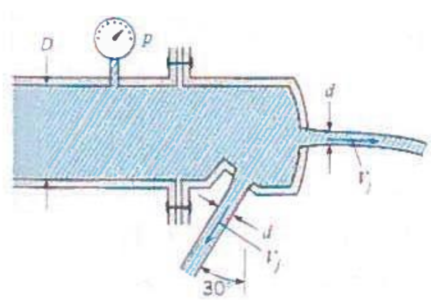
   a. Prove this equation satisfies continuity
   b. Calculate the acceleration of a fluid particle at point (r=4, θ =180°)
   c. Develop an equation for the fluid shear stress $\tau_{r\theta}$

   assume Newtonian

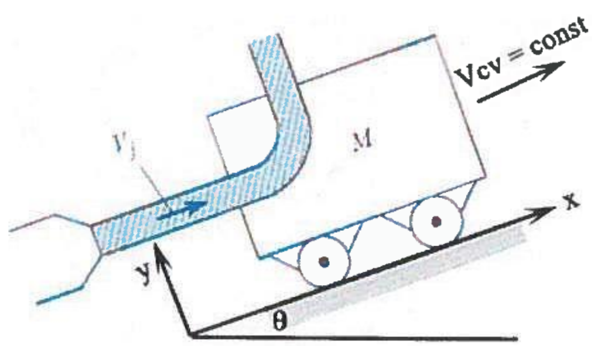
2. (10 marks) Find the force P on the parabolic gate shown in the figure given that H=2 m and the gate is 2 m wide. You can assume the gate has no mass and the water is at 20° C. 17 949.03N



3. (10 marks) The figure shows the spray head in a carwash. The spray head is attached to a 4-in diameter pipe (D=4 in) with an internal pressure of 45 psig. Two circular jets of 1-in diameter are directed towards the cars (d= 1 in). The velocity of each jet is 80 ft/s. If the jet head assembly, when full of water, has a mass of 0.2 lbm, what force is required at the flange to hold the spray head in place? Assume the water is at 80°F.



4. (8 marks) The cart in the figure has a mass (M) of 10 kg and is moved up an incline at a constant velocity by a jet of water (Vj=10 m/s) with a flow rate of 0.1 m^3/s. Assume the water has a density of 1000 kg/m^3 and that there is no frictional resistance to the cart. You may neglect the effect of gravity on the water and assume the mass of the water is much less than the mass of the cart. What is the angle (θ) of the incline if the velocity of the cart is constant at 2 m/s?



Midterm 2013

1) $u_r = (10 - 40/r^2)\cos\theta \qquad u_\theta = -(10 + 40/r^2)\sin\theta$

assume: incompressible

a) $\nabla \cdot \vec{v} = \frac{1}{r}\frac{\partial}{\partial r}(r u_r) + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \cancel{\frac{\partial u_z}{\partial z}}^{\,0} = 0$

$\frac{1}{r}\frac{\partial}{\partial r}\left[(10r - 40/r)\cos\theta\right] + \frac{1}{r}\frac{\partial}{\partial \theta}\left[-(10 + 40/r^2)\sin\theta\right] = 0$

$\frac{1}{r}(10 + 40/r^2)\cos\theta - \frac{1}{r}(10 + 40/r^2)\cos\theta = 0$

$0 = 0$

b) $a_r = \cancel{\frac{\partial u_r}{\partial t}}^{\,0} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + \cancel{u_z}^{\,0}\frac{\partial u_r}{\partial z}$

$= (10 - 40/r^2)\cos\theta\left(\frac{80}{r^3}\right)\cos\theta + \frac{(10 + 40/r^2)(\sin\theta)^2(10 - 40/r^2)}{r}$

$- \frac{(10 + 40/r^2)^2\sin^2\theta}{r}$

$= \frac{80}{4^3}(10 - 40/4^2)\cos^2 180 + \left[\frac{(10 + 40/4^2)(10 - 40/4^2) - (10 + 40/4^2)^2}{4}\right] \times \sin^2 180$

$= \boxed{9.375\, m/s^2}$

$a_\theta = \cancel{\frac{\partial u_\theta}{\partial t}}^{\,0} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + \cancel{u_z}^{\,0}\frac{\partial u_\theta}{\partial z}$

$= (10 - 40/r^2)\cos\theta\,(80/r^3)\sin\theta + \frac{(10 + 40/r^2)^2\sin\theta\cos\theta}{r}$

$- \frac{(10 - 40/r^2)(10 + 40/r^2)\cos\theta\sin\theta}{r}$

$= \left[(10 - 40/4^2)(80/4^3) + 1/4(10 + 40/4^2)^2 - 1/4(10 - 40/4^2)(10 + 40/4^2)\right]$
$\cos 180 \sin 180$

$= 0$

$a_z = 0$

$$x' = \frac{\tfrac{1}{2}(2)(1)^2 - \tfrac{1}{2}(1)^4}{1.33\,m^2} = 0.375\,m$$

find P using moment analysis

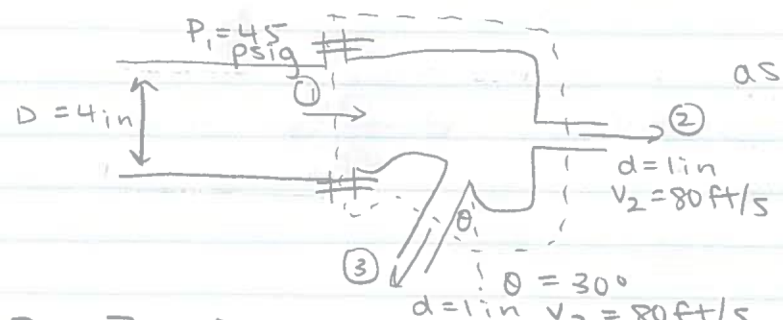
$$\sum M = P(H) - F_V(x') - F_H(H - y') = 0$$

$$P = \frac{F_V x' + F_H(H - y')}{H}$$

$$= \frac{(26\,107.68\,N)(0.375\,m) + (39\,161.52\,N)(2m - 1.33m)}{2\,m}$$

$$= \boxed{1.79 \times 10^5\,N}$$

3)



assume: - steady
- incompressibile
- inviscid

$P_2 = P_3 = 0$

$\rho = 1.93$ slug/ft^3 at 80°F

conservation of mass

$$Q_1 = Q_2 + Q_3$$

$$\bar{V}_1 A_1 = \bar{V}_2 A_2 + \bar{V}_3 A_3$$

$$V_1 = \frac{2(80\,ft/s)(\pi/4)(1\,in)^2}{(\pi/4)(4\,in)^2} = 10\,ft/s$$

conservation of momentum

$$\frac{\partial}{\partial t}\int_{cv} \cancel{\vec{V}\rho\, d\forall}^{\,0} + \int_{cs} \vec{V}\rho\vec{V}\cdot\hat{n}\, dA = \sum F_{sys}$$

$$x/\quad V_1\rho(-V_1)A_1 + V_2\rho V_2 A_2 - V_3 \sin 30\, \rho V_3 A_3 = -F_{Ax} + P_1 A_1$$

$$F_{Ax} = P_1 A_1 + V_1^2 \rho A_1 - V_2^2 \rho A_2 + V_3^2 \sin 30\, \rho A_3$$

$$= (45\,lbf/in^2)(\pi/4)(4in)^2 + (10\,ft/s)^2(1.93\,slug/ft^3)(1ft/12in)^2 (\pi/4)(1\,in)^2$$

$$+ (80\,ft/s)^2(1.93\,slug/ft^3)(1ft/12in)^2(\pi/4)(1\,in)^2(\sin 30 - 1)$$

$$= \boxed{532.86\,lbf}$$

Department of Chemical Engineering
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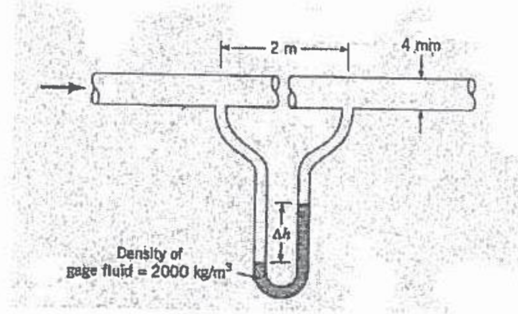
CHEE 314 Fluid Dynamics

Midterm Fall 2012

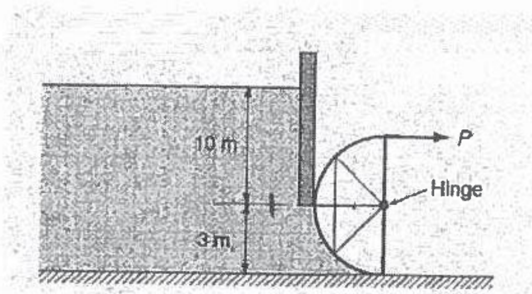
1. (8 marks) The velocity profile in the pipe is fully develop laminar flow in a straight pipe is given by the following equation. The fluid in the pipe is water at 5°C.

$$u(r) = \frac{1}{4\mu}\frac{dP}{dz}(r^2 - R^2)$$

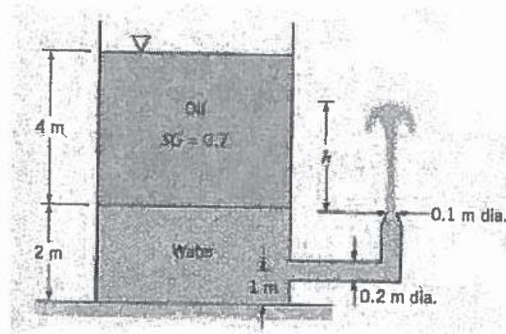


   a. Prove this is a possible velocity equation.
   b. What is the flow rate?
   c. What is the average velocity of water in the pipe?
   d. What is the wall shear stress in the pipe?

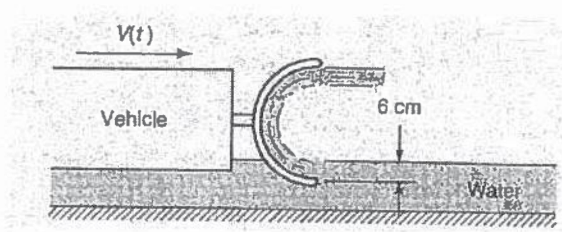
2. (10 marks) The gate in the figure is 5m wide (into the page) and weighs 400 N with a center of mass 0.9 m to the left of the hinge. Estimate the force P needed to open the gate.



3. (10 marks) A large tank contains a layer of oil floating on water as shown in the figure. The flow can be assumed steady and inviscid.
   a. Determine the height h the water will reach.
   b. What is the velocity of the water in the 0.2 m diameter pipe?
   c. What is the velocity of the water leaving the pipe? .



4. (12 marks) A vehicle with a mass of 5000 kg is traveling at 900 km/h. It is decelerated by lowering a 20cm wide scoop into water a depth of 6cm. If the water is deflected 180°, what distance must the vehicle travel to reduce its speed to 100 km/h.



Midterm 2012

1) $u(r) = \frac{1}{4\mu}\frac{dP}{dz}(r^2 - R^2)$

a) $\nabla \cdot u = 0$

$$\frac{\partial}{\partial z}\left(\frac{1}{4\mu}\frac{dP}{dz}(r^2 - R^2)\right) = 0 \checkmark$$

b) $$Q = \int_0^R \int_0^{2\pi} \frac{1}{4\mu}\frac{dP}{dz}(r^2 - R^2)\, r\, dr\, d\theta$$

$$= \int_0^R \frac{\pi}{2\mu}\frac{dP}{dz}(r^3 - rR^2)\, dr$$

$$= \left[\frac{\pi}{2\mu}\frac{dP}{dz}\left(\tfrac{1}{4}r^4 - \tfrac{1}{2}r^2R^2\right)\right]_0^R$$

$$= \frac{\pi}{2\mu}\frac{dP}{dz}\left(-\tfrac{1}{4}R^4\right) = \boxed{-\frac{\pi}{8\mu}\frac{dP}{dz}R^4}$$

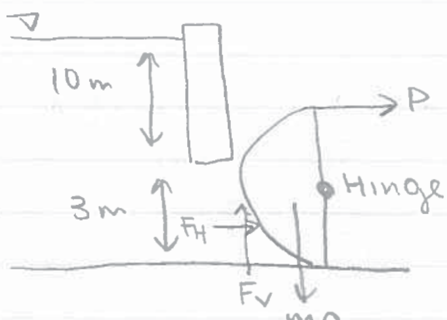
c) $$\bar{u} = \frac{Q}{A} = \frac{-\pi R^4/8\mu}{\pi R^2}\frac{dP}{dz}$$

$$\boxed{= -\frac{R^2}{8\mu}\frac{dP}{dz}}$$

d) $$\tau_{rz} = \mu\left[\frac{\partial u_z}{\partial r} + \cancelto{0}{\frac{\partial u_r}{\partial z}}\right]$$

$$= \mu\left(\frac{1}{4\mu}\right)\frac{dP}{dz}(2r) = \boxed{\frac{r}{2}\frac{dP}{dz}}$$

2)



assume: static, incompressible

width = 5 m

$F_H = \gamma h_c A = (998\ kg/m^3)(9.81\ m/s^2)(11.5m)(3m)(5m)$
$= 1\,688\,840\ N$

$y' = 2/3\ h = 2m$

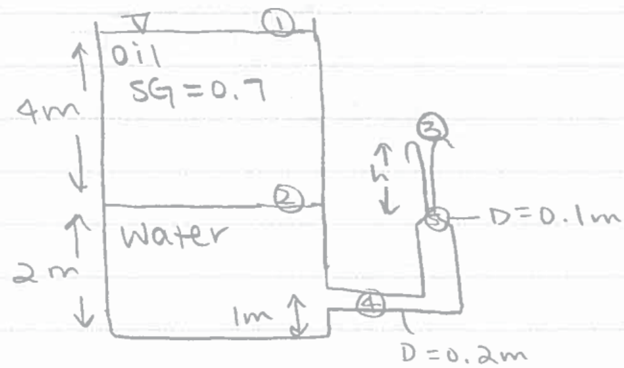
$F_V = \gamma \forall = (998\ kg/m^3)(9.81\ m/s^2)(\pi/4)(3m)^2(5m)$
$= 346\,020\ N$

$$x' = \frac{4R}{3\pi} = \frac{4(3m)}{3\pi} = 1.273m$$

$\Sigma M = P(3m) + F_V(1.273m) - F_H(2m) - (400N)(0.9m) = 0$

$$P = \frac{(1688840N)(2m) + (400N)(0.9m) - (346020N)(1.273m)}{3m}$$

$= 979\,158N \rightarrow \boxed{9.79 \times 10^5\ N}$

3) 

assume: steady
inviscid
large reservoir
incompressible

a) $P_2 = 0.7(998\ kg/m^3)(9.81\ m/s^2)(4m)$
$= 27\,413\ Pa$

$$\frac{P_2}{\gamma} + \cancel{\frac{V_2^2}{2g}}^{0} + \cancel{z_2}^{0} = \cancel{\frac{P_3}{\gamma}}^{0} + \cancel{\frac{V_3^2}{2g}}^{0} + z_3$$

$$z_3 = h = \frac{(27\,413\ Pa)}{(998\ kg/m^3)(9.81\ m/s^2)} = \boxed{2.8m}$$

b) $V_4 A_4 = V_5 A_5$

$$V_5 = V_4 \frac{A_4}{A_5} = V_4 \frac{\pi/4\ D_4^2}{\pi/4\ D_5^2} = V_4 \frac{D_4^2}{D_5^2}$$

$$\frac{P_2}{\gamma} + \cancel{\frac{V_2^2}{2g}}^{0} + \cancel{z_2} = \cancel{\frac{P_5}{\gamma}}^{0} + \frac{V_5^2}{2g} + \cancel{z_5}$$

$$\frac{P_2}{\gamma} = \frac{1}{2g}\left(V_4 \frac{D_4^2}{D_5^2}\right)^2$$

$$V_4 = \sqrt{\frac{2gP_2}{\gamma} \frac{D_5^2}{D_4^2}}$$

$$= \sqrt{\frac{2(9.81\,m/s^2)(27\,413\,Pa)}{(9.81\,m/s^2)(998\,kg/m^3)} \frac{(0.1m)^2}{(0.2m)^2}}$$

$$= \boxed{1.85\,m/s}$$

c) $V_5 = (1.85\,m/s)\dfrac{(0.2m)^2}{(0.1m)^2} = \boxed{7.41\,m/s}$

4)

reduce speed 900 km/h to 100 km/h
scoop 20cm wide

assume: incompressible, water is static

$$0 = \frac{\partial}{\partial t}\int_{cv} \cancel{\vec{W}\rho\, d\forall}^{\,0} + \frac{\partial}{\partial t}\int_{cv} \vec{V}_{cv}\,\rho\, d\forall + \int_{cs} \vec{W}\rho\,\vec{W}\cdot\hat{n}\, dA$$

↖ acceleration term

$$-M_{cart}\frac{\partial \vec{V}_{cv}}{\partial t} = -W_1\rho(-W_1)A_1 + W_2\rho W_2 A_2$$

C of mass: $Q_1 = Q_2$

since $A_1 = A_2$ $\quad W_1 = W_2$

$$-M\frac{\partial \vec{V}_{cv}}{\partial t} = 2W^2\rho A \qquad\qquad \frac{du}{dt} = \frac{du}{dx}u$$

$$W = V - V_{cv} = -V_{cv}$$

$kg\,m/s^2$ $\quad -M\dfrac{\partial \vec{V}_{cv}}{\partial t} = 2V_{cv}^2\rho A = -M\dfrac{\partial \vec{V}_{cv}}{\partial x}\vec{V}_{cv}$

$$\int_{900}^{100}\frac{\partial V_{cv}}{V_{cv}} = \int_0^x -\frac{2\rho A}{M}\,dx$$

$$\Big[\ln V_{cv}\Big]_{900}^{100} = -\frac{2\rho A}{M}x$$

$$x = -\frac{(5000\,kg)}{2(998\,kg/m^3)(0.06m)(0.20m)}\ln\left(\frac{100\,km/h}{900\,km/h}\right) = \boxed{459\,m}$$

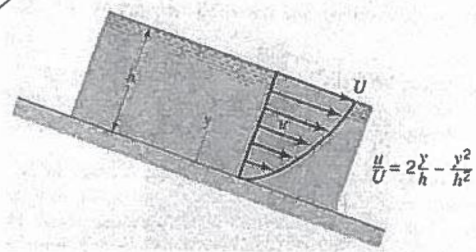
Department of Chemical Engineering
McGill University

CHEE 314 Fluid Dynamics

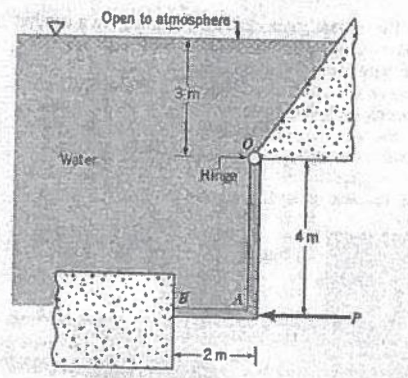
Midterm — Fall 2011

glycerin?

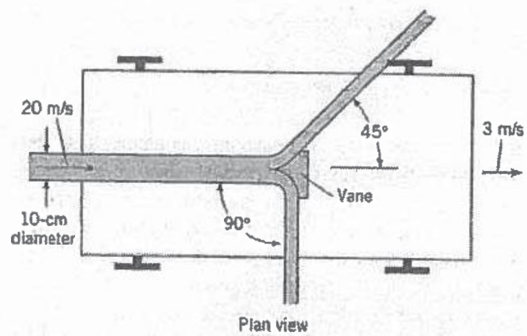
1. (5 marks) A layer of glycerin at 60°C flows down an inclined fixed surface with the velocity profile shown in the figure. What is the shear stress that the water exerts on the fixed surface if U=3 m/s and h=0.1 m? What is the shear stress at the surface of the glycerin?



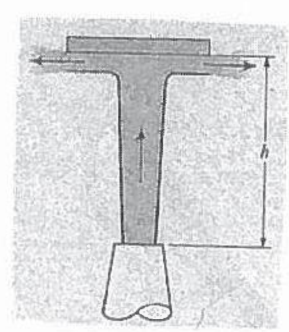
2. (8 marks) The rigid gate OAB, is hinged at O and rests against a rigid support at B. What horizontal force P needs to be applied to hold the gate closed if the gate is 3m wide? Neglect the weight of the gate. The back of the gate is open to atmosphere and the water is at 10°C.



3. (10 marks) A vane on a moving cart deflects a 10-cm water jet as shown. The jet has a velocity of 20 m/s and the cart moves at a constant speed of 3 m/s. If the vane splits the jet equally so half goes one way and the other have the other, what force is exerted on the vane by the jet? Assume the jet of water is at 10°C and the cart is frictionless.



4. (12 marks) A vertical jet of water (10°C) leaves a nozzle with a velocity of 10 m/s and a diameter of 20 mm. It is noticed that the jet diameter increases with height. A plate having a mass of 1.5 kg is placed on the jet and is suspended (i.e. does not move). A) What is the vertical distance h from the jet exit to the bottom of the plate? B) What is the diameter of the jet just below the plate? **Do not** assume gravity has no effect on the jet, but **you may** assume the mass of the water is negligible. **HINT:** first find an expression for the change in velocity with height.



Midterm 2011

1)

$U = 3 m/s$

$h = 0.1 m$

assume: Newtonian fluid
- no slip
- SG glycerin = 1.26
- incompressible

$\tau = \mu \frac{du}{dy}$ $\frac{u}{U} = \frac{2y}{h} - \frac{y^2}{h^2}$

$\frac{du}{dy} = U\left(\frac{2}{h} - \frac{2y}{h^2}\right)$

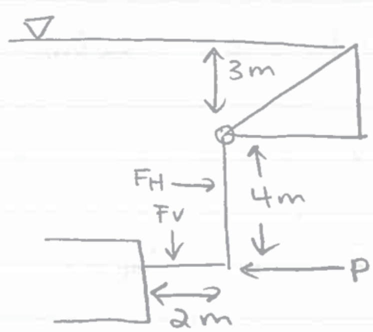
$\tau = \mu U\left(\frac{2}{h} - \frac{2y}{h^2}\right)$

$\mu = 1 \times 10^{-1}\ N \cdot s/m^2$ for glycerin at 60°C

$\tau = (1 \times 10^{-1}\ N \cdot s/m^2)(3 m/s)\left(\frac{2}{0.1m} - \frac{2(0m)}{(0.1m)}\right)$

$= \boxed{6\ Pa}$

2)



assume - static
- incompressible
- mass gate = 0

3 m wide

$F_H = \gamma_{H_2O} h_c A$

$= (998 kg/m^3)(9.81 N/kg)(5m)(4m)(3m)$

$= 5.88 \times 10^5 N$

$y' = \frac{I_{\hat{x}\hat{x}}}{y_c A} + y_c$

$= \frac{1/12 (3m)(4m)^3}{(5m)(3m)(4m)} + 5m = 5.267m$

$F_V = \gamma \forall$

$= (998 kg/m^3)(9.81 N/kg)(2m)(3m)(7m)$

$= 4.11 \times 10^5 N$

$x' = 1m$

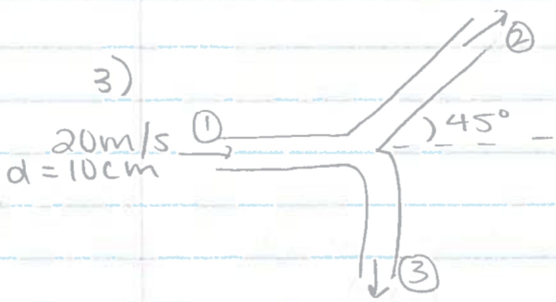
Moment analysis

$\sum M = F_H(y' - 3m) + F_V x' - P(4m) = 0$

$P = \dfrac{(5.88 \times 10^5 N)(5.267m - 3m) + (4.11 \times 10^5 N)(1m)}{4m}$

$= \boxed{4.36 \times 10^5 N}$

3)



cart moves 3m/s

assume: - steady flow
- incompressible
- no effect gravity
- mass + friction cart negligible

C of mass:

$V_1 A_1 = V_2 A_2 + V_3 A_3$

jet splits equally in half so $V_2 A_2 = V_3 A_3$

Bernoulli:

$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$

neglect changes in P + Z so $V_1 = V_2$

$V_1 A_1 = 2V_2 A_2$

$A_1 = 2A_2 \rightarrow A_2 = A_3 = \frac{1}{2}A_1$

C of momentum:

x/ $w_1 \rho(-w_1)A_1 + w_1(\cos 45)\rho w_1(\frac{1}{2}A_1) = F_x$

$F_x = (1000 kg/m^3)(\pi/4)(0.10m)^2(17m/s)^2(\frac{1}{2}\cos 45 - 1)$

on jet $= -1467N \rightarrow \boxed{1467N \text{ on vane}}$

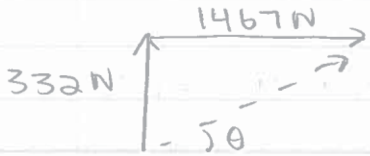
y/ $-w\rho w(\frac{1}{2}A_1) + w(\sin 45)\rho w(\frac{1}{2}A_1) = F_y$

$F_y = (1000 kg/m^3)(\pi/4)(\frac{1}{2})(0.10m)^2(17m/s)^2(\sin 45 - 1)$

$= -332N \rightarrow \boxed{332N \text{ on vane}}$

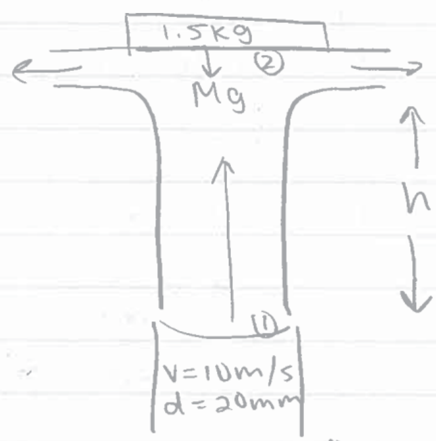
net force

$$F = \sqrt{1467^2 + 332^2} = \boxed{1504\ N}$$



$$\theta = \tan^{-1}\left(\frac{332}{1467}\right) = \boxed{12.8°}$$ above horizontal

4)



assume: steady
- incompressible
- inviscid

$$\cancelto{0}{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + z_2 = \cancelto{0}{\frac{P_1}{\gamma}} + \frac{V_1^2}{2g} + \cancelto{0}{z_1}$$

$$V_2^2 = V_1^2 - 2gh$$

$$V_2 = \sqrt{V_1^2 - 2gh}$$

C of momentum:

y/ $V_2 \rho (-V_2) A = -Mg$

$$-V_2^2 \rho A = -Mg$$

$$(V_1^2 - 2gh) \rho (\pi/4) d_2^2 = Mg$$

C of mass:

$$V_1 (\pi/4) d_1^2 = V_2 (\pi/4) d_2^2$$

$$d_2 = \sqrt{\frac{V_1}{V_2}}\, d_1$$

plug into above

$$(V_1^2 - 2gh) \rho (\pi/4) \frac{V_1}{\sqrt{V_1^2 - 2gh}} d_1^2 = Mg$$

$$\sqrt{V_1^2 - 2gh} = \frac{4Mg}{\pi \rho V_1 d_1^2}$$

$$v_1^2 - 2gh = \left(\frac{4Mg}{\pi \rho v_1 d_1^2}\right)^2$$

$$v_1^2 - \left(\frac{4Mg}{\pi \rho v_1 d_1^2}\right)^2 = 2gh$$

$$h = \frac{1}{2g}\left[v_1^2 - \left(\frac{4Mg}{\pi \rho v_1 d_1^2}\right)^2\right]$$

$$= \frac{1}{2(9.81\,m/s^2)}\left[(10\,m/s)^2 - \left(\frac{4(1.5\,kg)(9.81\,m/s^2)}{\pi(1000\,kg/m^3)(10\,m/s)(0.02\,m)^2}\right)^2\right]$$

$$\boxed{= 3.98\,m}$$

$$v_2 = \sqrt{(10\,m/s)^2 - 2(9.81\,m/s^2)(3.98\,m)}$$

$$= 4.68\,m/s$$

$$d_2 = \sqrt{\frac{10\,m/s}{4.68\,m/s}}\,(20\,mm) = \boxed{29.2\,mm}$$

$\frac{ft}{s^2} \times \frac{lbf\, s^2}{ft \cdot ft^3} \times ft^3$

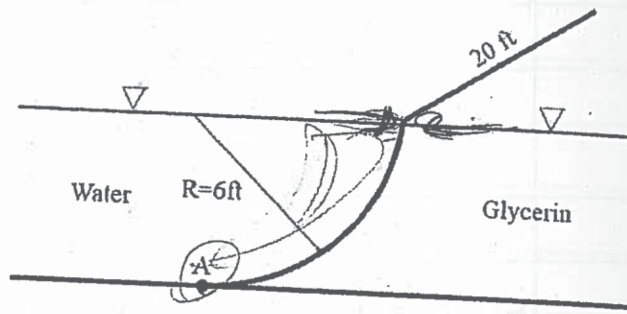
Department of Chemical Engineering
McGill University

CHEE 314 Fluid Dynamics

Midterm

Fall 2010

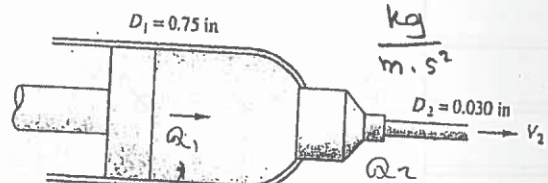
1. (10 marks) A curved gate, 20ft long, is used to separate a reservoir of water and glycerin. If the gate is hinged at point A, what horizontal force would be required at the top of the gate (liquid surface) to hold the gate in place? You can assume the liquids are at 68°F.



$\frac{kg}{m^3} \times \frac{m^2}{s^2}$

$\frac{kg}{m \cdot s^2}$

2. (15 marks) A hypodermic needle contains a liquid serum (SG=1.05, $\mu = 2.0 \times 10^{-3}\ N \cdot s/m^2$). The serum is injected steadily at 6 cc per second (cm^3/s). Q



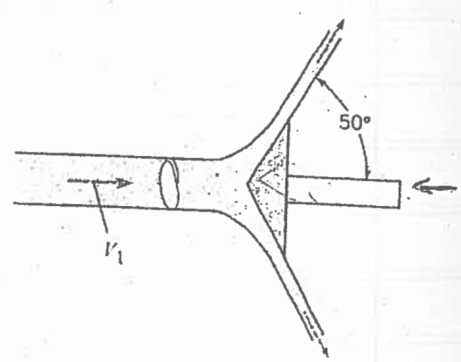
$\mu = 2.0 \times 10^{-3}\ N \cdot s/m$

$Q_1 = 6\ cm^3/s$

   a. How fast does the plunger advance?
   b. Assuming there are no losses, use Bernoulli's equation to estimate the pressure inside the chamber (D_1) if the serum is injected to the atmosphere?
   c. Using the conservation of momentum, estimate the force needed to move the plunger. You may need to make some additional assumptions and simplifications. Compare your answer to the force you would estimate by Bernoulli's equation. Do they agree, if not why?
   d. What is the maximum shear stress on the serum in the needle (D_2)? You may assume the liquid is Newtonian and the flow is fully developed with a velocity profile of $V_2(r) = 2\bar{V}_2\left(1 - \left(\frac{r}{R_2}\right)^2\right)$, where $\bar{V}_2$ is the average velocity.

$\frac{N \cdot s}{m^2} \times \frac{m/s}{m} \Rightarrow N/m^2$

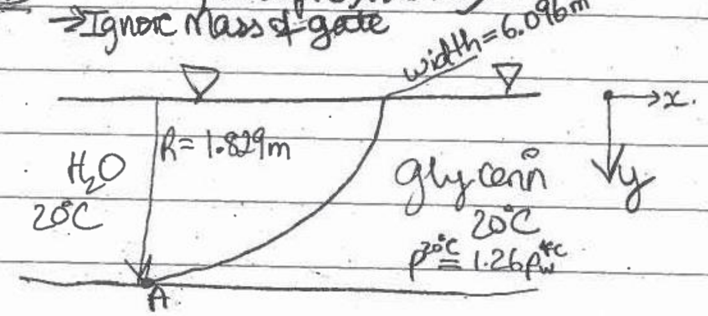
3. (8 marks) A steady jet of water with a velocity of 25 m/s (V_1) is deflected by a wedge (50°) that is moving steadily at 13 m/s towards the jet. The diameter of the jet is 10 cm. Determine the external horizontal force needed to move the cone. You may neglect the effect of gravity and assume the water is at 4°C.

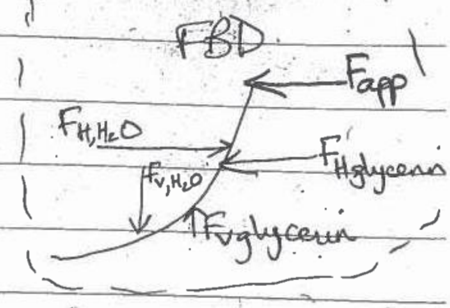


Question 1

I converted to SI units

Assumptions → Static, incompressible fluids
→ Ignore Mass of gate





FOR WATER

$$F_{H,H_2O} = \gamma A_{cproj} h_{cproj}$$

Projection is ←6.096m→ 1.829m [rectangle] centroid $= h/2 = \frac{1.829}{2}$

$$= \rho_w g[(1.829)(6.096)]\left(\frac{1.829}{2}\right)$$

$\frac{0\times 9}{5}+32$ $F_{H,H_2O} = 99.825\text{ KN}$ $\qquad \rho_w^{20°C} = 998$

$$y'_{H_2O} = y_{cproj} + \frac{I_{\hat{x}\hat{x}}}{A_{cproj}\, y_{cproj}} = \frac{1.829}{2} + \frac{\frac{1}{12}(\cancel{6.096})(1.829)^{\cancel{3}2}}{\cancel{(6.096)(1.829)}\left(\frac{1.829}{2}\right)}$$

$$y' = \frac{1.829}{2} + \frac{1}{12}\left(\frac{1.829}{1/2}\right) = 1.2193\text{ m}$$

$$F_V = \rho g \forall_{displaced} = (998)(9.81)\left(\frac{\pi}{4}(1.829)^2\right)(6.096)$$

$$F_V = \rho g A w$$

$$F_V = 156.805\text{ KN}$$

$$X'_{H_2O} = \frac{4R}{3\pi} = 0.776\text{ m}$$

FOR GLYCERIN

$\gamma_{gly} = SG_{gly}\,\rho_w^{4°C} g$

$$F_{H,glycerin} = \overbrace{\gamma_{gly} A_{cproj} h_{cproj}}^{\text{same as for } H_2O \text{ side}} = (1.26)(1000)(9.81)(1.829)^2(6.\ldots)\frac{}{2}$$

$$F_{H,glycerin} = 126.032\text{ KN}$$

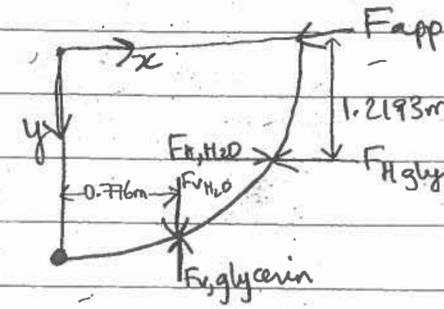
$$y'_{glycerin} = y_{cproj} + \frac{I_{\hat{x}\hat{x}}}{A_{cproj}\, y_{cproj}} = 1.2193\text{ m} \quad (\text{Same as } H_2O \text{ side}$$

$$F_V\ glycerin = \rho_{gly} g V_{displaced} = (1.26)(1000)(9.81)\left(\frac{\pi}{4}(1.829)^2\right)(6.096)$$

$$F_V\ glycerin = 197.971\text{ KN}$$

Centroid of displaced area! $\quad X'_{gly} = \frac{4R}{3\pi} = 0.776\text{m}$ } Same as for H_2O side

Thus, as F_H and F_V for both H_2O & glycerin act through same locations.



$$\sum_{\substack{\text{about}\\ A}} \text{Moments} = 0$$

Resultant Horizontal force

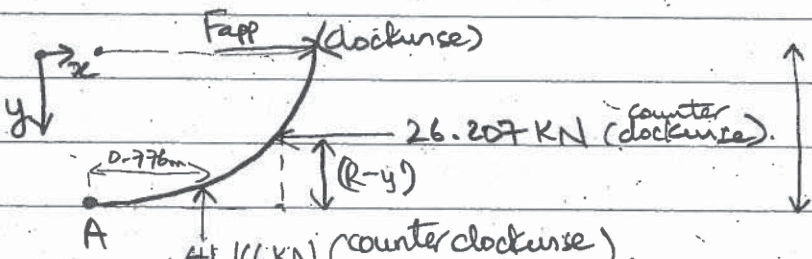
$$F_{H,R} = F_{H,H_2O} - F_{H,gly} = (99.825 - 126.032)\,10^3\,N$$
$$= -26.207\,KN \text{ (to the left) i.e., } -x \text{ direction}$$

Resultant Vertical Force

$$F_{V,R} = \underbrace{F_{V,H_2O}}_{\text{weight}} - \underbrace{F_{V\,gly}}_{\text{Buoyancy}} = (156.805 - 197.971)\times10^3$$

$F_{V,R} = -41.166\,KN$ (Upwards, -ve y-direction in my drawing)

Thus FBD becomes



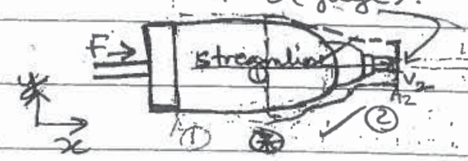
$$\sum_{\text{about } A} M_o = 0 \Rightarrow (41.166\times10^3)(0.776) + (26.207\times10^3)(1.829 - 1.219) - F_{app}(1.829) =$$

$$\therefore F_{app} = \frac{47931}{1.829} = 26.206\text{ kN in } +x \text{ direc}$$

ANSWER $F_{app} = 26.21\,KN \equiv 5890\,lbf$

10

(2) 1 Assuming (i) Steady Flow
(ii) Incompressible liquid
(iii) No accumulation in CV until ✱



(a)

$D_1 = 0.01905\ m$

Conservation of Mass in CV before ✱

$D_2 = 0.000762\ m$

$$Q_{in} = Q_{out}$$
$$\bar{V}_1 A_1 = \bar{V}_2 A_2 = \dot{Q}$$

$$\dot{Q} = A_1\bar{V}_1 \quad \therefore \quad \bar{V}_1 = \frac{(6\times10^{-6})\,m^3/s}{\pi\left(\frac{0.01905}{2}\right)^2 m^2} = 0.02105\ ms^{-1} \equiv 0.829\ in/s$$

4

$\dot{Q} = A\frac{dx}{dt}$ ∴ Plunger advances 0.02105 m per second to the right.
or 0.829 inch per second

Bernoulli's.

$P_2 = 0$ (guage)

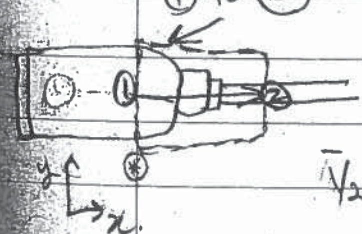
(b) $$\frac{P_1}{\rho} + \frac{V_1^2}{2} + \cancel{gz_1} = \cancel{\frac{P_2}{\rho}}^{\,0} + \frac{V_2^2}{2} + \cancel{gz_2}$$ $z_1 = z_2$ (same height)

along stream line 1 to 2

$$\therefore \quad P_1 = \frac{\rho}{2}(\bar{V}_2^2 - \bar{V}_1^2)$$

7

$$\boxed{P_1 = \frac{(1.05)(1000)}{2}\left(13.16^2 - 0.02105^2\right) = \underline{90.866\ kPa}\ \text{(guage)}}$$



$$\bar{V}_2 = \frac{\dot{Q}}{A_2} = \frac{(6\times10^{-6})}{\pi\left(\frac{0.000762}{2}\right)^2} = 13.16\ ms^{-1}$$

(c) Cons. momentum (No accumulation in CV before ✱)

x-direction $$\int_{CS} \vec{V}_x \rho(\vec{V}\cdot\hat{n})\,dS = \sum F_{sys} = \cancel{\sum F_B}^{\,0} + \sum F_{sys}.$$

$P_2 = 0$ guage

$$[\rho V_1(-V_1A_1)] + [\rho V_2(V_2A_2)] = P_1A_1 - \cancel{P_2A_2}^{\,0} + \ldots$$

$$\rho(V_2^2A_2 - V_1^2A_1) = \underbrace{P_1A_1}_{\text{applied Force}}$$

$P_1A_1 = (1.05)(1000)(6\times10^{-6})(13.16 - 0.0210\ldots$

$$\therefore P_1 = \frac{0.082775}{\pi\left(\frac{0.01905}{2}\right)^2} = 290 \text{ Pa. (gauge)}$$

3

This does not agree with Bernoulli's result of $P_1 = 90866$ Pa
$\simeq 313$ times smaller $\rightarrow$ Bernoulli's ignores frictional losses which seem to be significant ~~the difference~~ in this case, hence

(d) $\rightarrow$ Newtonian Liquid : $\tau = \mu \frac{dV}{dr}$
$\rightarrow$ Fully developed flow :

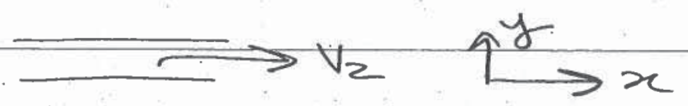
$$\frac{dV_z}{dr} = \frac{d}{dr}\left(2\bar{V}_2\left(1 - \frac{r^2}{R_2^2}\right)\right) = 2\bar{V}_2 \frac{d}{dr}\left(1 - \frac{r^2}{R_2^2}\right) = 2\bar{V}_2\left(\frac{-2r}{R_2^2}\right)$$

$$\therefore \tau = \frac{-\mu \cdot 4r\bar{V}_2}{R_2^2}$$

Maximum shear stress occurs when $r = R_2$

$$\therefore \tau_{max} = -\frac{\mu \cdot 4 R_2 V_2}{R_2^2} = \frac{-4\mu \bar{V}_2}{R_2}$$

$$\tau_{max} = \frac{-4 \times (2\times10^{-3})(13.16)}{\left(\frac{0.000762}{2}\right)} = -276 \text{ Pa.}$$



Then shear on fluid is in the $-x$ direction (i.e. to the left).

4

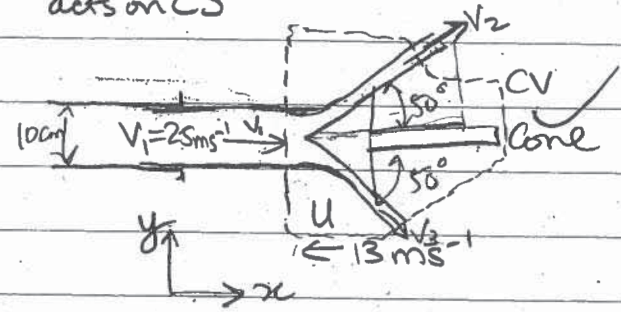
Question 3

Assumptions (i) Steady, incompressible, fully developed flow

(v) H_2O @ $4^\circ C$, $\therefore \rho_{H_2O} = 1000\,kg/m^3$ (ii) Neglecting effect of gravity

(iii) Ignoring mass of cone and water in CV (i.e. ignoring weight).

(iv) Only $P_{atm} = 0$ guage acts on CS



$w_1 = V_1 - V_{cv}$

$w_2 = V_2 - V_{cv}$

$w_3 = V_3 - V_{cv}$

Cons. of Mass: V_{cv} = constant $\therefore \dfrac{\partial V_{cv}}{\partial t} = 0$

No accumulation in CV $\therefore$ $\int_{CS} \rho(\vec{w} \cdot \hat{n}\, dS) = 0$

$$-w_1 A_1 + w_2 A_2 + w_3 A_3 = 0$$

$$\therefore \quad w_1 A_1 = w_2 A_2 + w_3 A_3.$$

Bernouilli's from ① → ② $V_1 = V_2$ $\therefore$ $W_1 = W_2$ $\therefore$

similarly $V_1 = V_3$ $\quad W_1 = W_3$ $\therefore$ $W_2 = W_3$.

Neglecting g, then $A_1 = A_2 + A_3 = 2A_2 \therefore A_2 = A_3$

Cons of Momentum: x-direction no accumulation & steady flow

$$\int_{CS} \vec{W}_x \rho(\vec{W} \cdot \hat{n})\, dS = \sum F_{sys,x} = \sum F_{B,x} + \cancel{\sum F_{surface,x}}^{\,0}$$

$\Rightarrow$ $w_1 \rho(-w_1 A_1) + \rho w_2 \cos(50^\circ)(w_2 A_2) + \rho w_3 \cos(50^\circ)(w_3 A_3) = F_{app,x}$

$$\vec{w}_1 = \vec{V}_1 - \vec{V}_{cv} = 25 - (-13) = 38\ ms^{-1}$$

Because $w_2 = w_3$ and $A_1 = A_2 + A_3$.

$$\rho\left(w_2^2 \cos(50^\circ)(A_2 + A_3) - w_1^2 A_1 \right) = F_{app,x}$$

$\Rightarrow$ $\rho A_1 (w_2^2 \cos(50^\circ) - w_1^2) = F_{app,x}$

44 PTO

$\rho A_1 w_1^2 (\cos(50^\circ) - 1) = F$ ... $F = -(1000)(\pi/4)$ ...

Thus the applied force is $F_{app\,x} = 4.051$ KN in the $-x$ direction

i.e. 4.051 KN to the left

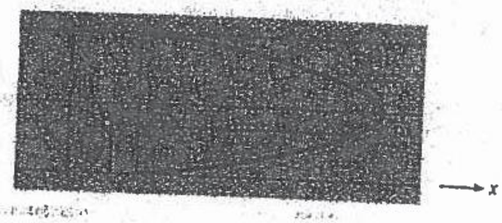
8

CHEE 314 Fluid Dynamics

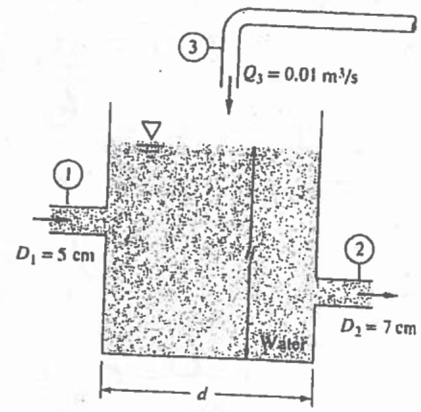
Midterm

Fall 2007

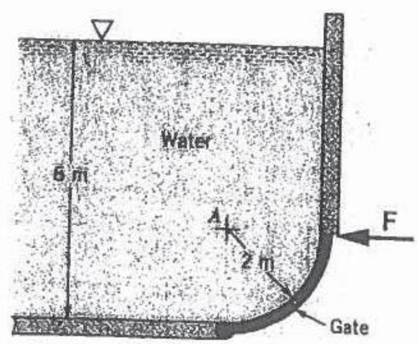
1. (8 marks) The velocity distribution for flow between two parallel walls is $u=100y(0.1-y)$ ft/s, where y is measured in ft and the distance between the walls is B=0.1 ft. The velocity distribution is constant across the width (W) of the channel.
   a. What is the flow rate?
   b. What is the average velocity?
   c. What is the shear stress at the wall if the fluid is water at 68°F?
   d. Show this is a possible incompressible flow distribution.



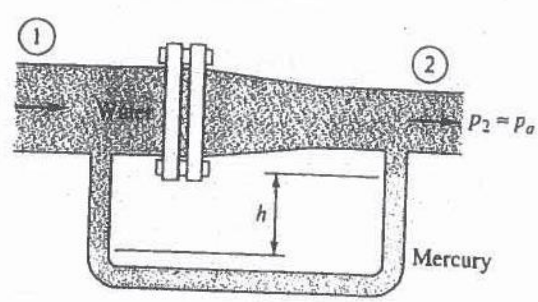
2. (6 marks) A tank is filled with water at 20°C from a pipe as shown in the figure.
   a. Derive an expression for the water-level change dh/dt in terms of the flows (Q_1, Q2 and Q3) and the diameter of the tank d.
   b. If the water level h is constant, determine the exit velocity V_2 if the average velocity V_1=3 m/s.



3. (10 marks) A 3-m-long curved gate is located in the side of a reservoir containing water (20°C). The curved gate is pinned at the bottom of the reservoir. What force (F) needs to be applied at the side of the gate to keep it closed.



4. (8 marks) The pipe restriction shown in the figure his held together by a flange. The diameter at entrance is D_1=8 cm and at the exit D_2=5 cm. The pressure at the exit can be assumed to be atmospheric (P_2=Pa). All fluids are at 20°C. If V_1= 5m/s and the manometer reading is h=58 cm, estimate the total force on the flange.



1.a) $Q = \int_{cs} \vec{V} \cdot \hat{n}\, dS$

$Q = \int_0^W dz \int_0^{B=0.1} 100y(0.1-y)\,dy$

$Q = W\int_0^{0.1} (10y - 100y^2)\,dy$

$Q = W\left[5y^2 - \frac{100y^3}{3}\right]_0^{0.1} = W\left[5(0.1)^2 - \frac{100(0.1)^3}{3} - 0\right]$

$Q = \frac{W}{60}\frac{ft^3}{s} \approx 0.0167\,W\,\frac{ft^3}{s}$

Assume
→ incompressibl
→ steady
→ fully developd
→ no slip

b) $\bar{V} = \frac{Q}{A} = \frac{\frac{W}{60}\frac{ft^3}{s}}{(0.1\,ft)(W\,ft)} = \frac{\cancel{W}}{(60)(0.1)(\cancel{W})}\frac{ft}{s}$

$\bar{V} = \frac{1}{6}\frac{ft}{s} \approx \underline{0.167\ ft/s}$

c) $\tau = \mu \cdot \frac{du}{dy}$

need μ_{H_2O} @ 68°F = 20°C

from Figure A.2, $\mu_{H_2O}(20°C) = 1\times10^{-3}\ \frac{N\cdot s}{m^2}$

$\tau = \mu_{H_2O}\frac{d}{dy}\left[10y - 100y^2\right] = \mu_{H_2O}(10 - 200y)$

@ the wall, $y = 0\ ft = 0\ m$

$\tau = \left(\frac{1\times10^{-3}\ N\cdot s}{m^2}\right)\left[(10 - 200(0))\ s^{-1}\right] = 0.01\ N/m^2$

no units of length, so no conversion necessary

$\underline{\tau = 0.01\ Pa}$

$\tau = (2.1\times10^{-5}\quad)(10 - 200y)|_{y=0}$

$f \approx 2.1\times10^{-4}\ lb_f/ft^2$

d) for an incompressible fluid flow field $\Rightarrow \nabla \cdot \vec{V} =$

$\vec{V} = u\hat{i} + v\hat{j} = 100y(0.1-y)\hat{i} + 0\hat{j}$ (no vertical comp of flow)

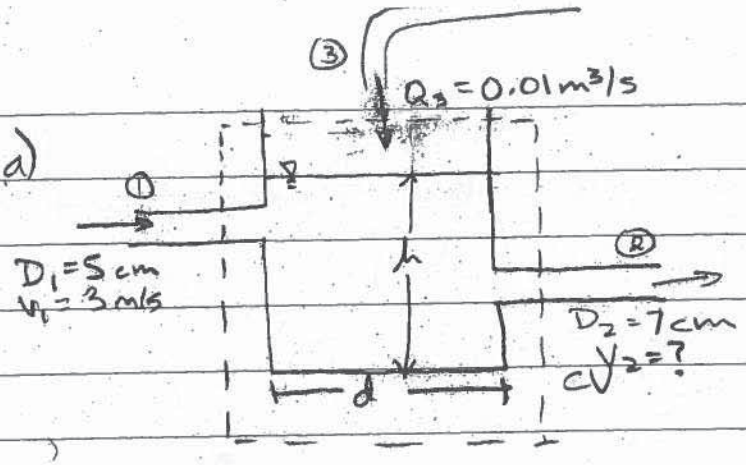
$$\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial}{\partial x}\left[100y(0.1-y)\right] + \frac{\partial}{\partial y}[0]$$

$$\nabla \cdot \vec{V} = 0$$

$\Rightarrow$ This is a possible incompressible flow distributio

7

2. a)



Assume
→ steady flow
→ incompressible flu

Conservation of Mass

$$\frac{d}{dt}\int_{cv} \rho \, d\forall + \int_{cs} \rho (\vec{v}\cdot\hat{n}) dA = 0$$

$$\rho \frac{d}{dt}\int_{cv} d\forall + \rho \int_{cs} (\vec{v}\cdot\hat{n}) dA = 0 \quad \text{(incompressible} \Rightarrow \rho = \text{const)}$$

$$\frac{d}{dt}[\forall] + (-v_1)A_1 + V_2 A_2 - Q_3 = 0$$

$$\frac{d}{dt}[hA] = Q_1 - Q_2 + Q_3$$

$$\frac{dh}{dt} A + h \cancelto{0}{\frac{dA}{dt}} = Q_1 - Q_2 + Q_3$$

$$\frac{dh}{dt} = \frac{Q_1 - Q_2 + Q_3}{A} = \frac{Q_1 - Q_2 + Q_3}{\frac{\pi}{4} d^2}$$

$$\underline{\frac{dh}{dt} = \frac{4(Q_1 - Q_2 + Q_3)}{\pi d^2}}$$

b) h is constant $\Rightarrow \frac{dh}{dt} = 0$

$$\text{so} \quad \frac{4(Q_1 - Q_2 + Q_3)}{\pi d^2} = 0$$

$$4(Q_1 - Q_2 + Q_3) = 0$$

$$v_1 A_1 - V_2 A_2 + Q_3 = 0$$

$$v_1 A_1 + Q_3 = V_2 A_2$$

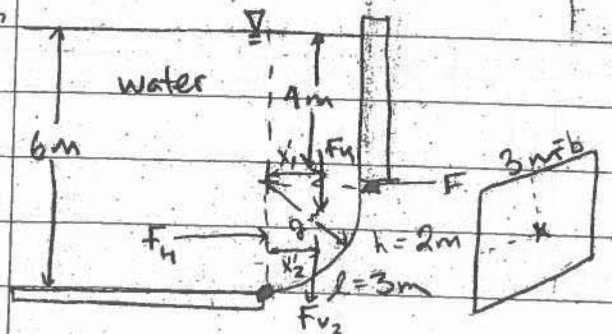
$$V_2 = \frac{v_1 A_1 + Q_3}{A_2} = \frac{(3\,m/s)(\frac{\pi}{4})(5\times10^{-2}\,m)^2 + (0.01\,m^3/s)}{(\frac{\pi}{4})(7\times10^{-2}\,m)^2}$$

$$V_2 = \left(\frac{0.01589\ \frac{m^3}{s}}{3.848\times10^{-3}\ m^2}\right)$$

$$\underline{V_2 = 4.13\,m/s}$$

6

3.



Assume
→ ~~static~~
→ incompressible
→ mass of the gate is negligible

$$F_H = \gamma h_{c\,proj} A_{proj} = (998\ \tfrac{kg}{m^3})(9.81\ \tfrac{N}{kg})(4\,m + 1\,m)(2m)(3m)$$

$$F_H = 293.7\ kN$$

$$y' = \frac{I_{\hat{x}\hat{x}}}{A_{proj}\, y_{c\,proj}} + y_{c\,proj} = \frac{\frac{1}{12}\cancel{b}h^{\cancel{3}2}}{(\cancel{b}h)\, y_{c\,proj}} + y_{c\,proj} = \frac{1\,(2m)^2}{12\,(4m+1m)} + (4m + 1m)$$

$$y' = 5.067\ m$$

$$F_V = \underset{\text{from prism}}{F_{V_1}} + \underset{\text{from}}{F_{V_2}}$$

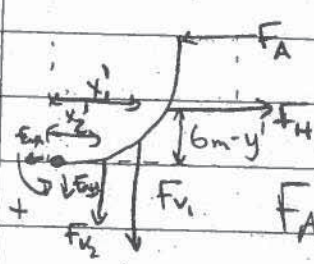
$$F_{V_1} = \gamma \forall_1 = (998\ \tfrac{kg}{m^3})(9.81\ \tfrac{N}{kg})(4m)(2m)(3m) = 234.97\ kN$$

$$x_1' = \frac{2\,m}{2} = 1\,m$$

$$F_{V_2} = \gamma \forall_2 = (998\ \tfrac{kg}{m^3})(9.81\ \tfrac{N}{kg})(\tfrac{1}{4}\pi)(2m)^2(3m) = 92.27\ kN$$

$$x_2' = \frac{4r}{3\pi} = \frac{4(2m)}{3\pi} = 0.849\ m$$

GATE FBD



$$\Sigma M_0 = 0$$

$$-F_{V_2} x_2' - F_{V_1} x_1' - F_H(6 - y') + F_A r = 0$$

$$F_A r = F_{V_2} x_2' + F_{V_1} x_1' + F_H(6 - y')$$

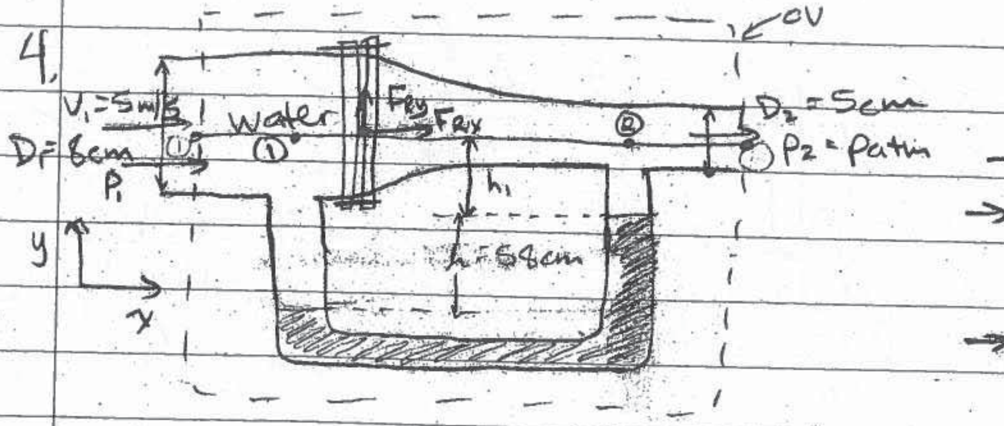
$$F_A = \frac{(92.27kN)(0.849m) + (234.97kN)(1m) + (293.7kN)(0.933)}{2m}$$

$$F_A = 294\ kN$$

10

(50)

4.



Assume
→ steady flow
→ incompressible fluids (H_2O, H
→ neglect gravity

① conservation of mass

$$\cancel{\frac{\partial}{\partial t}}\int_{cv} \rho\, d\forall + \int_{cs} \rho(\vec{V}\cdot\hat{n})\,dA = 0$$

0 because of incompressibility

$$\int_{s_1} \rho(\vec{V}\cdot\hat{n})\,dA + \int_{s_2} \rho(\vec{V}\cdot\hat{n})\,dA = 0$$

$$\cancel{\rho}(-V_1)A_1 + \cancel{\rho}(V_2)A_2 = 0$$

$$V_1 A_1 = V_2 A_2 \Rightarrow V_2 = \frac{V_1 A_1}{A_2} = \frac{(5\,m/s)\cancel{(\frac{\pi}{4})}(8\,cm)^2}{\cancel{(\frac{\pi}{4})}(5\,cm)^2}$$

$$V_2 = 12.8\,m/s$$

② ~~use~~ manometer to find P_1

$$P_1 = \cancel{P_2} + \cancel{h_1\gamma_w} + h\gamma_{Hg} - h\gamma_w - \cancel{h_1\gamma_w}$$

$P_2 \to 0$ (gauge P)

$$P_1 = h(\gamma_{Hg} - \gamma_w) = h\,\rho_{H_2O,4^\circ C}\, g\,(SG_{Hg} - SG_{H_2O})$$

$$P_1 = (58\times10^{-2}\,m)(1000\,\tfrac{kg}{m^3})(9.8\,\tfrac{N}{kg})(13.55 - 0.998)$$

$$P_1 = 71.35\,kPa$$

(???)

③ conservation of momentum

$$\cancel{\frac{\partial}{\partial t}}\int_{cv} \vec{V}\rho\, d\forall + \int_{cs} \vec{V}\rho(\vec{V}\cdot\hat{n})\,dA = \sum F_{system}$$

0 steady flow

x direction: $V_1\rho(-V_1)A_1 + V_2\rho(V_2)A_2 = F_{Rx} + P_1 A_1$

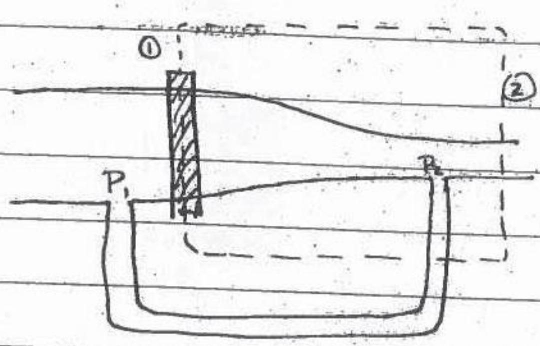
$$-\rho V_1^2 A_1 + \rho V_2^2 A_2 - P_1 A_1 = F_{Rx}$$

$$F_{Rx} = -(998\,\tfrac{kg}{m^3})(5\,m/s)^2(\tfrac{\pi}{4})(8\times10^{-2}\,m)^2 + (998\,\tfrac{kg}{m^3})(12.8\,\tfrac{m}{s})^2(\tfrac{\pi}{4})(5\times10^{-2}\,m)^2$$

$F_{Rx} = -125.41\,N + 321.06\,N - 358.62\,N$

$\underline{F_{Rx} = -163\,N}$

8



→ assume pressure at flanc
is equal to P_1 and that
pressure at outlet is P_2

→ assume manometer is sta

→ manometer accounts for losses due to a change in geometry

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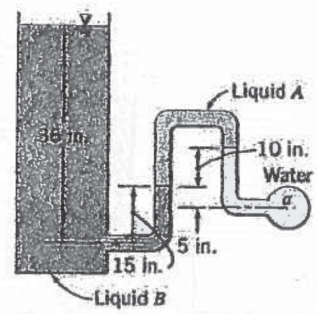
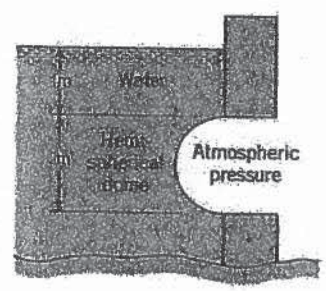
CHEE 314 Fluid Dynamics

Midterm Fall 2006

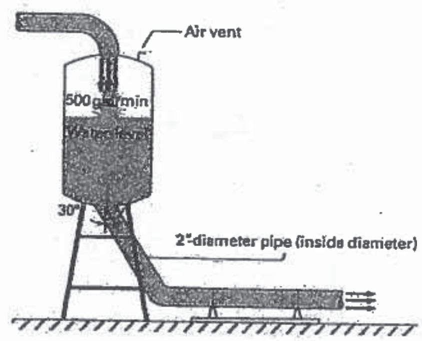
1) Determine the gage pressure in psi at point a, if liquid A has a SG=0.80 and liquid B has SG=1.10. The liquid at point a is water and the tank is open to the atmosphere. You can assume the temperature is 4°C.



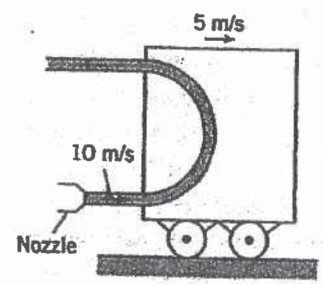
2) What is the magnitude, direction and location of F_H and F_V on the hemispherical dome window in the large tank shown in the figure. The dome is 2.0m in diameter and starts 1.0m below the surface of the water. Assume the temperature of the water is 10°C.



3) Water is being added to a storage tank at the rate of 500 gal/min. Water also flows out of the bottom through a 2.0 in (inside diameter) pipe with an average velocity of 60 ft/sec. The inside diameter of the storage tank is 10.0 ft. Find the rate at which the water level is rising or falling.

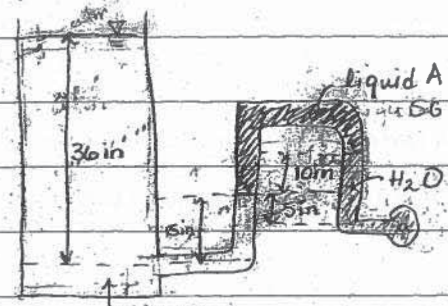


4) Water is used to accelerate a cart as shown. The flow rate of the jet is 0.1 m^3/s and the velocity is 10 m/s. When the water hits the cart it is deflected 180°. The mass of the cart is 10 kg and the density of the water 1000 kg/m^3. The mass of water in the jet is much less than the cart. Calculate the acceleration of the cart when the velocity is 5 m/s.



Mid-Term 2006 Answers

1.



Assume

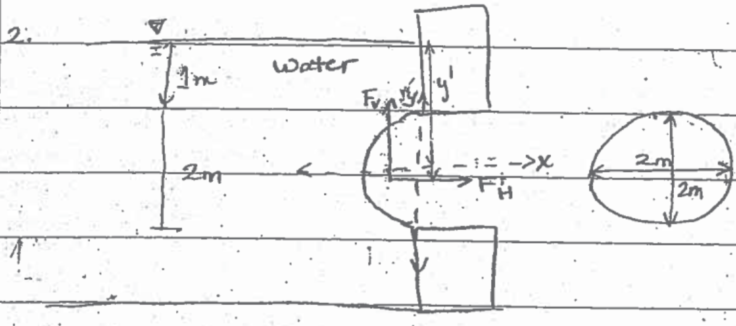
- Static
- Incompressible
- T = 4°C

$P_A - 15\,in\,\gamma_W + 10\,in\,\gamma_A - 21\,in\,\gamma_B = \cancel{P_B}^{\,0}$ $\qquad P_B = 0$, atmospheric

$P_A - \frac{15\,in}{12\,in}ft\left(1.94\,\frac{lbf \cdot s^2}{ft^4}\right)\left(32.2\,ft/s^2\right) + \frac{10\,in}{12\,in}ft\,(0.80)(1.94)(32.2) - \frac{21\,in}{12\,in}ft\,(1.10)(1.94)(3$

$P_A = 156.69\,\frac{lb_f}{ft^2}\left(\frac{1\,ft}{12\,in}\right)^2 = \boxed{1.088\ psi_{gage}}$

2.



Assume

- Static
- Incompressible
- Neglect atmospheric P
- T = 10°C

$F_V = \gamma \forall$

$\forall_{hemisphere} = \frac{1}{2}\frac{4}{3}\pi r^3 = \frac{2}{3}\pi (1m)^3 = \frac{2\pi}{3}$

$F_V = (1000\ kg/m^3)(9.8\ m/s^2)(\frac{2}{3}\pi)m^3$

$\underline{F_V = 20{,}525\ N}$ ⟶ Thru the centroid $\bar{x} = \frac{4r}{3\pi} = \frac{4(1)}{3\pi} = 0.424m$ left of y axis

$F_H = \gamma h_{c\,proj} A_{proj}$ $A_{proj} = \pi r^2 = \pi\ m^2$

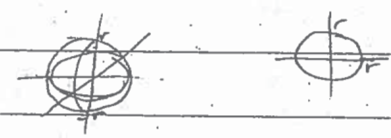
$F_H = (1000\ kg/m^3)(9.8\ m/s^2)(\pi)(2)\ m^3$ $h_{c\,proj} = 2m = y_{c\,proj}$

$\underline{F_H = 61{,}575\ N}$

$y' = \frac{I_{xx}}{A_{proj}\, y_{c\,proj}} + y_{c\,proj}$

$y' = \frac{\frac{1}{12}\cancel{b}h^3}{\cancel{bh}(2m)} + 2m$

$y' = \frac{\frac{1}{12}(2m)^2}{2m} + 2m = 2.167m - 2m = .167$ below x axis



6

$\int_0^{2\pi} d\theta \int_0^r r\,dr \int_{-r}^{r} dz$

$2\pi\ 2r^2\,dr$

$2\pi\, 2\frac{r^3}{3}$

$\frac{4\pi}{3}r^3$

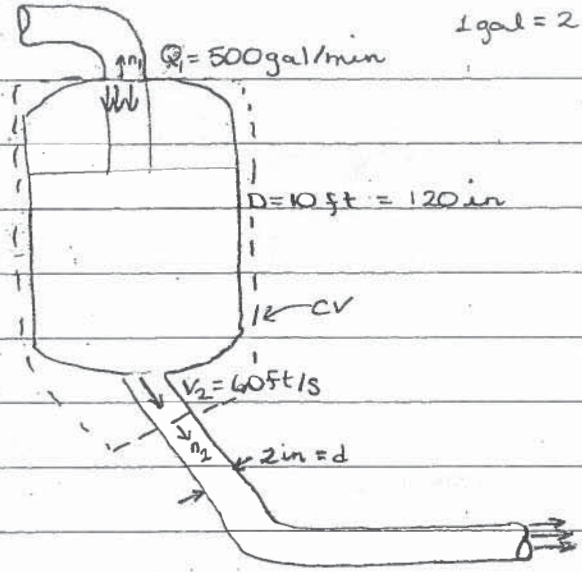
The F_V = 20,525N and acts in the positive y-direction at x = -0.424m

The F_H = 61,575 N and acts in the positive x direction at the position y = -0.167m.

(see axes on diagram)

$1\,gal = 231\,in^3$

3.



Assume

- ~~Incompressible~~
- 1-D Uniform flow
- neglect gravity

$$Q_1 = 500\,\frac{gal}{min} \times \frac{231\,in^3}{1\,gal} \times \frac{1\,min}{60\,s} = 1925\,in^3/s$$

$$V_2 = 60\,ft/s \times \frac{12\,in}{1\,ft} = 720\,in/s$$

C of Mass

$$\frac{d}{dt}\int_{cv} \rho\, d\forall + \int_{cs} \rho(\vec{v}\cdot\hat{n})\, dA = 0$$

$$\frac{d\forall}{dt} - Q_1 + V_2 A_2 = 0$$

$$\frac{d\forall}{dt} = -V_2 A_2 + Q_1 \qquad \frac{d\forall}{dt} = \frac{dh}{dt} A_1$$

$$\frac{dh}{dt} = \frac{-V_2 A_2 + Q_1}{A_1}$$

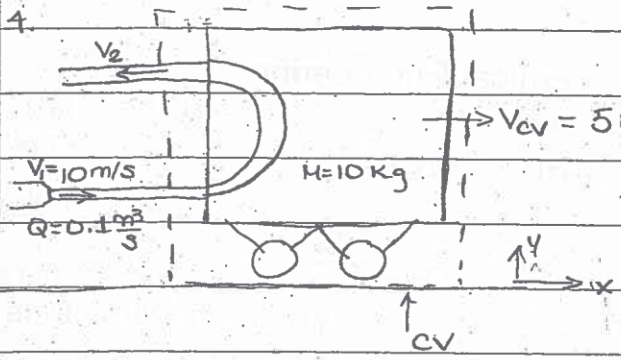
10

$$\frac{dh}{dt} = \frac{-(720\,in/s)\left(\frac{\pi}{4}(2^2\,in^2)\right) + 1925\,in^3/s}{\frac{\pi}{4}(120^2\,in^2)}$$

$$\frac{dh}{dt} = -0.0298\,in/s$$

i.e the water level is falling by 0.0298 inches every second.

4.



Assume

- Incompressible
- Steady
- 1-D uniform flow
- Neglect mass of water
- Neglect friction on surface.

$\vec{W} = \vec{V} - \vec{V}_{cv}$

$\vec{W} = 10 - 5 = 5\,m/s$

$Q = VA$

$\frac{0.1\,\frac{m^3}{s}}{10\,m/s} = A$

$0.01\,m^2 = A$

Conservation of Mass

$$\underbrace{\cancel{\frac{d}{dt}\int_{cv} \rho\, d\forall}}_{0,\ \text{steady}} + \int_{cs} \rho (\vec{W} \cdot \hat{n})\, dA = 0$$

$Q_1 = Q_2$

$A_1 = A_2 \quad \therefore \quad W_1 = W_2 = 5\,m/s$

Conservation of Momentum

$$\underbrace{\cancel{\frac{d}{dt}\int_{cv} \vec{W}\rho\, d\forall}}_{0\ \text{steady}} + \int_{cs} \vec{W}\rho(\vec{W}\cdot\hat{n})\, dA = \underbrace{\cancel{\sum \vec{F}_{sys}}}_{0\ \text{no friction},\ P_1A_1 = P_2A_2} - \int_{cv} \frac{\partial \vec{V}_{cv}}{\partial t}\rho\, d\forall$$

x dir

$$W_1\rho(-W_1)A_1 - W_2\rho(W_2)A_2 = -M_{cart}\frac{\partial \vec{V}_{cv}}{dt}$$

$$-W_1^2\rho A_1 - W_2^2\rho A_2 = -M_{cart}\frac{\partial \vec{V}_{cv}}{dt}$$

$$\frac{-2W^2\rho A}{-M_{cart}} = \frac{\partial \vec{V}_{cv}}{\partial t}$$

$$\frac{2(5^2\,m^2/s^2)(0.01\,m^2)(1000\,\cancel{kg}/m^3)}{10\,\cancel{kg}} = \frac{\partial \vec{V}_{cv}}{dt}$$

$$50\,m/s^2 = \frac{\partial \vec{V}_{cv}}{dt}$$

10

The acceleration of the cart is $50\,m/s^2$

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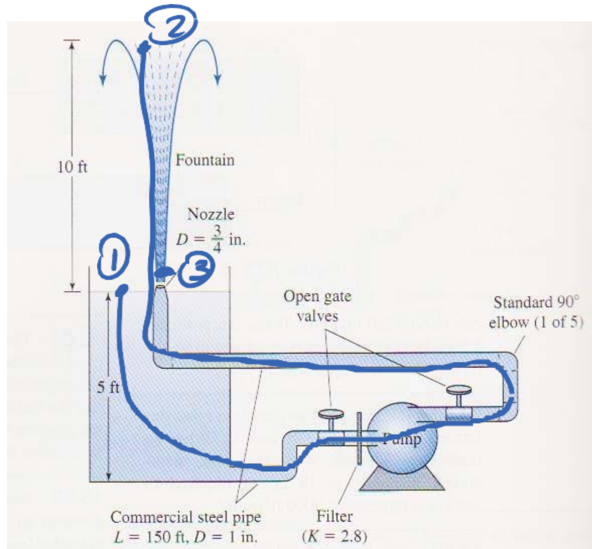
CHEE 314 Fluid Dynamics

Quiz 2 Fall 2013

1. **(15 marks)** A pump is used in a large reservoir to create a fountain. There is a 150 ft of commercial steel pipe of diameter 1-in and 5 standard 90° elbows. A filter (K=2.8) and two open gate valves are placed around the pump. Assume the water is at 68°F and the nozzle is a sudden contraction. What head (ft) needs to be added to the system to create a 10 ft high water

Assume
- steady
- incomp
- large reservoir



feature?

Energy (3) → (2)

$$\frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

0 0 0

$$V_2 = \sqrt{2g(z_3 - z_2)} = 25.4 \text{ ft/s}$$

$$A_1 = \frac{\pi(\frac{1}{12})^2}{4} = 0.0055 \text{ ft}^2$$

$$A_2 = \frac{\pi(.75/12)^2}{4} = 0.0031 \text{ ft}^2$$

$$\therefore Q = V_2 \times A_2 = 25.4 \times 0.0031 = 0.0779 \text{ ft}^3/\text{s} \rightarrow \approx 35 \text{ gpm}$$

$$\therefore V_3 = Q/A_1 = 14.27 \text{ ft/s} = V$$

$$\therefore Re_1 = \frac{VD}{\nu} = \frac{14.27(1/12)}{1.08 \times 10^{-5}} = 1.101 \times 10^5 \leftarrow \text{turbulent}$$

$$\epsilon/D = \frac{0.00015}{1/12} = 0.0018$$

Bernoulli's 1 - 3

$$\cancel{\frac{P_3}{\gamma}} + \frac{V_3^2}{2g} + \cancel{z_3} = \cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} - H_L - H_S$$

0 0 0 0

$$H_S = -H_L - \frac{V_3^2}{2g}$$

H_L

Major → from moody $f \simeq 0.024$

Colebrook → 0.02443

Minor elbows → $\frac{L_e}{D} = 30 \times 5$ entrance $k = 0.5$

gate valve $\frac{L_e}{D} = 8 \times 2$

filter $k = 2.8$

Sudden Contraction $AR = \left(\frac{.75}{1}\right) = 0.5625$

fig 8.15 $k \simeq 0.2$

$$\therefore H_L = 0.024 \frac{V^2}{2g}\left(\frac{150}{1/12} + 30 \times 5 + 8 \times 2\right) + \frac{V^2}{2g}(2.8 + 0.5) + \frac{V_2^2}{2g}(0.2)$$

$= 161.7$ ft

$$\therefore H_S = -\left(161.7 + \frac{V_3^2}{2g}\right)$$

$= -(171.8)$ ft

$\therefore$ in Hp → $\mathcal{P} = \gamma Q H_S$

$= 62.4 \times Q \times (171.8)$ $\frac{\text{lbf} \cdot \text{ft}}{s}$

$= \frac{834.9}{550} \simeq 1.5$ hp

Department of Chemical Engineering
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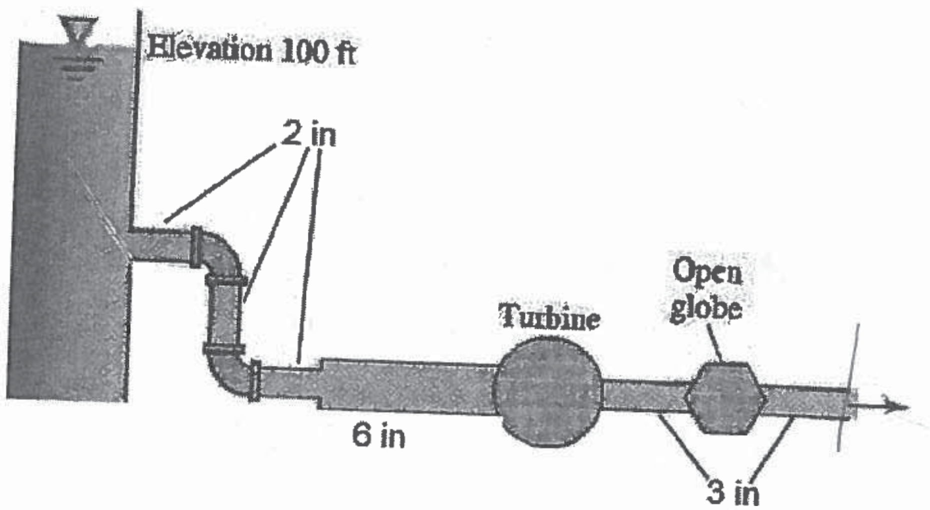
CHEE 314 Fluid Dynamics

Quiz 2

Fall 2012

1. **(8 marks)** A sphere of diameter D and density ρ_s falls through a liquid of density ρ and viscosity μ. If the Reynolds number is small (Re<0.4) show that the viscosity (μ) of the liquid can be calculated knowing the terminal viscosity of the ball (U), diameter (D) and density of the liquid and ball (ie. derive an equation).

2. **(12 marks)** In the piping system shown, all pipes are made of cast iron and there are 125 ft of 2-in pipe, 75 ft of 6-in pipe and 150 ft of 3-in pipe. There are 2 standard 90° elbows and one open globe valve. The pipe exit is 100 ft below the surface of a large reservoir. What is the power (Hp) that can be extracted by the turbine if the flow rate is 0.16 ft^3/s and the water is at 68°F?



2 QUIZ 2

) Sphere D, ρ_s $\qquad Re = \frac{U D \rho_\ell}{\mu}$

liquid ρ, μ

Show $\mu = f(U, D, \rho_s, \rho_\ell)$

$\mu = \frac{M}{LT}$ $\quad U = \frac{L}{T}$ $\quad D = L$ $\quad \rho_s = \frac{M}{L^3}$ $\quad \rho_\ell = \frac{M}{L^3}$

repeat U, D, ρ_ℓ

$\Pi_1 = \mu U^a D^b \rho_\ell^c \Rightarrow \frac{M}{LT}\left(\frac{L}{T}\right)^a (L)^b \left(\frac{M}{L^3}\right)^c$

$M: 1 + c = 0 \rightarrow c = -1$

$T: -1 - a = 0 \rightarrow a = -1$

$L: -1 + a + b - 3c = 0 \quad -1 - 1 + b + 3 \quad b = -1$

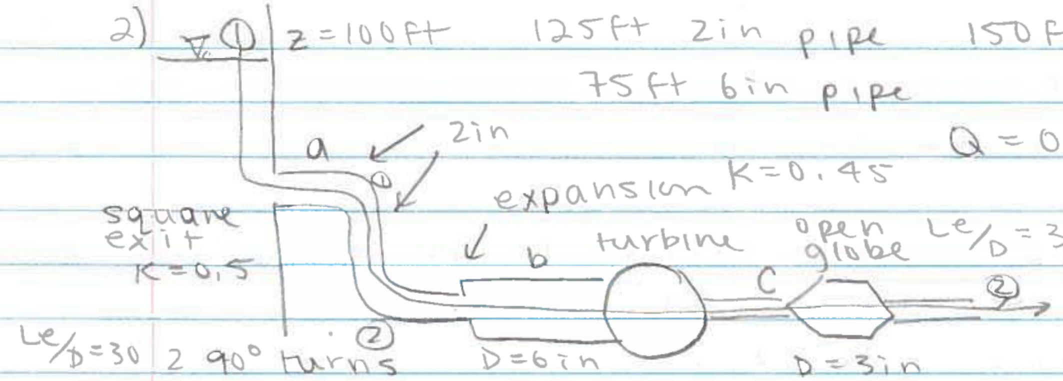
$\Pi_1 = \frac{\mu}{U D \rho_\ell} = \frac{1}{Re}$

$\Pi_2 = \frac{\rho_s}{\rho_\ell}$

$\boxed{\frac{1}{Re} = f\left(\frac{\rho_s}{\rho_\ell}\right)}$ $\qquad$ Should you go further?

2) $z = 100\,ft$ 125 ft 2 in pipe   150 ft 3 in pipe

75 ft 6 in pipe



$Q = 0.16\,ft^3/s$

$T = 68°F$

$\nu = 1.08 \times 10^{-5}\,ft^2/s$

$\rho = 1.94\,slug/ft^3$

cast iron $\varepsilon = 0.26\,mm$

find power in pump

(A) $V_a = \dfrac{0.16\,ft^3/s}{(\pi/4)(2/12\,ft)^2} = 7.33\,ft/s$

$Re_a = \dfrac{(7.33\,ft/s)(2/12\,ft)}{1.08 \times 10^{-5}\,ft^2/s} = 113\,176.8484$

$\dfrac{\varepsilon}{D_a} = \dfrac{(0.26\times10^{-3}\,m)(1\,in/0.0254\,in)}{2\,in} = 0.005\,118\,11$

(B) $f_a = 0.0315$

(C) $V_b = \dfrac{0.16\,ft^3/s}{(\pi/4)(6/12\,ft)^2} = 0.8149\,ft/s$

$Re_b = \dfrac{(0.8149\,ft/s)(6/12\,ft)}{1.08\times10^{-5}\,ft^2/s} = 37\,725.616\,14$

$\dfrac{\varepsilon}{D_b} = \dfrac{(0.26\times10^{-3}\,m)(1\,in/0.0254)}{6\,in} = 0.001\,706\,036$

(D) $f_b = 0.0267$

(E) $V_c = \dfrac{0.16\,ft^3/s}{(\pi/4)(3/12\,ft)^2} = 3.26\,ft/s$

$Re_c = \dfrac{(3.26\,ft/s)(3/12\,ft)}{1.08\times10^{-5}\,ft^2/s} = 75\,451.232\,28$

$\dfrac{\varepsilon}{D_c} = \dfrac{(0.26\times10^{-3}\,m)(1\,in/0.0254\,in)}{3\,in} = 0.003\,412\,073$

(F) $f_c = 0.028\,701\,161$

Hilroy

$$\cancel{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + \cancel{z_2} = \cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + z_1 - H_L - H_s$$

$$H_s = -\frac{V_2^2}{2g} + z_1 - H_L$$

$$H_L = 0.5\frac{V_a^2}{2g} + 2(30)f_a\frac{V_a^2}{2g} + 0.45\frac{V_a^2}{2g} + 340 f_c\frac{V_c^2}{2g} + f_a\frac{L_a}{D_a}\frac{V_a^2}{2g} + f_b\frac{L_b}{D_b}\frac{V_b^2}{2g} + f_c\frac{L_c}{D_c}\frac{V_c^2}{2g}$$

$$= 26.63\text{ ft}$$

$$H_s = -\frac{(3.26\text{ ft/s})^2}{2(32.174\text{ ft/s}^2)} + 100\text{ ft} - 26.63\text{ ft}$$

$$= 73.20\text{ ft}$$

$$P = (73.20\text{ ft})(1.94\text{ slug/ft}^3)(32.174\text{ ft/s}^2)(0.16\text{ ft}^3/\text{s})$$

$$= 731\text{ lbf}\cdot\text{ft/s} \Rightarrow \boxed{1.33\text{ hp}}$$

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CHEE 314 Fluid Dynamics

Quiz 2 Fall 2011

1. **(10 marks)** The wall shear stress τ_w in a boundary layer is assumed to be a function of the upstream velocity (U), boundary layer thickness (δ), local turbulence velocity (u'), density (ρ) and local pressure gradient (dp/dx). Find a set of dimensionless parameters to reduce the number of variables. Use ρ, U and δ as the repeating variables.

2. **(12 marks)** Your friend asks you to help size a pump for their cottage. The cottage is 120 ft above the surface of the lake. The water (68°F) will need to travel through 2000 ft of 6 in diameter cast iron pipe and exit to atmosphere. A flow rate of 3 ft^3/s is needed at the exit. You may neglect minor losses (DO NOT neglect major losses).
   a. Assume the pump is 75% efficient, what horsepower pump is needed?
   b. The temperature of the lake drops to 40°F in the winter. Do you think the pump will still be able to achieve the required flow rate? Justify your answer.

① $\tau_w = f(U, \delta, U', \rho, \frac{dP}{dx})$ $n = 6$

$\tau_w = \frac{M}{LT^2}$; $U = L/T$; $\delta = L$; $U' = L/T$; $\rho = \frac{M}{L^3}$; $\frac{dP}{dx} = \frac{M}{L^2T^2}$

# of dimentions $(r) = 3$

- using Buckinham-Π method: # of terms $= 6 - 3 = 3$
- repeating varibles: (ρ, U, δ)

* $\therefore \Pi_1 = C_{\tau w} = \tau_w (\rho)^a (U)^b (\delta)^c = \left(\frac{M}{LT^2}\right)\left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b (L)^c$

$(M) \to 1 + a = 0$
$(L) \to -1 - 3a + b + c = 0$
$(t) \to -2 - b = 0$

$\boxed{a = -1 \quad b = -2 \quad c = 0}$

$$\boxed{\Pi_1 = \frac{\tau_w}{\rho U^2}}$$

* $C_{U'} = \Pi_2 = (U')(\rho)^a (U)^b (\delta)^c = \left(\frac{L}{T}\right)\left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b (L)^c$

$L \to 1 - 3a + b + c = 0$
$M \to a = 0$
$t \to -1 - b = 0$

$\boxed{a = 0 \quad b = -1 \quad c = 0}$

$$\boxed{\Pi_2 = \frac{U'}{U}}$$

* $\Pi_3 = C_{\frac{dP}{dx}} = \left(\frac{dP}{dx}\right)(\rho)^a (U)^b (\delta)^c = \left(\frac{M}{L^2t^2}\right)\left(\frac{M}{L^3}\right)^a \left(\frac{L}{T}\right)^b (L)^c$

$M \to 1 + a = 0$
$L \to -2 - 3a + b + c = 0$
$t \to -2 - b = 0$

$\boxed{a = -1 \quad b = -2 \quad c = 1}$

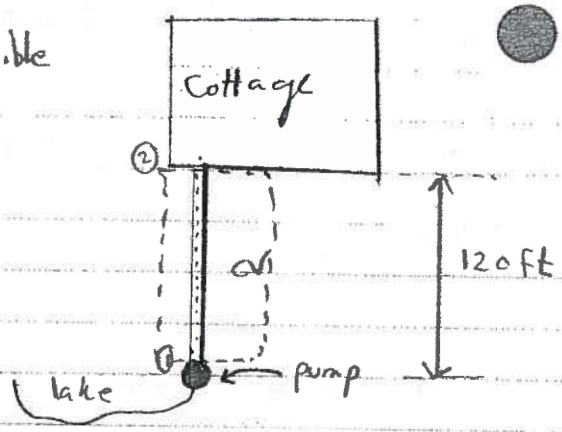
10

$$\boxed{\Pi_3 = \frac{dP}{dx} \frac{\delta}{\rho U^2}}$$

$$\boxed{\text{Find: } \frac{\tau_w}{\rho U^2} = f\left(\frac{U'}{U}, \frac{dP}{dx} \frac{\delta}{\rho U^2}\right)}$$

(2) Assume :-

1) steady 2) Uniform ($\alpha = 1$) 3) incompressible
4) $T = 68^\circ F$ 5) Neglect Minor losses
6) stream line from (1) – (2)



info:-

- @ $68^\circ F$: $\rho = 1.94 \ \text{slugs}/ft^3$
  $\mu = 2.10 \times 10^{-5} \ \frac{lbf \cdot s}{ft^2}$
- $\varepsilon / D = 0.0017$ [Cast iron]
- $L = 2000 \ ft$, $D = 0.5 \ ft$, $Q = 3 \ ft^3/s$

Solution:-

- Average velocity $(\bar{V}) = Q/A = \dfrac{3}{\left(\frac{0.5}{2}\right)^2 \pi} = 15.28 \ ft/s$
- $Re = \dfrac{\rho V D}{\mu} = \dfrac{(1.94)(15.28)(0.5)}{(2.10 \times 10^{-5})} = 7.1 \times 10^5$ (turbulent)
- Now Bernoulli (1) → (2)

continuity

pump Not in CV

$$\frac{P_2}{\gamma} + \cancel{\frac{V_2^2}{2g}}^{\,0} + Z_2 = \frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + Z_1 - \cancel{H_s} - H_L$$

$$P_1 - P_2 = (Z_2 + H_L)\,\gamma = \gamma\left(Z_2 + f \frac{L}{D} \frac{\bar{V}^2}{2g}\right)$$

from moody charts:- $f = 0.0228$

$$P_1 - P_2 = (1.94)(32.174)\left(120 + (0.0228)\left(\frac{2000}{0.5}\right)\left(\frac{15.28^2}{(2)(32.174)}\right)\right)$$

$$\Delta P = 28145 \ lbf/ft^2 = H_s$$

$$\text{power} = \frac{(H_s)}{(\gamma)}(\gamma)(Q) = (28145)(3) = 84433.6 \ ft \ lbf/s$$

efficiency $= 0.75$ ∴ power $= \dfrac{84433.6}{0.75} = 112578 \ \dfrac{ft \ lbf}{s} = 205 \ hp$.

$\frac{37.73}{40}$

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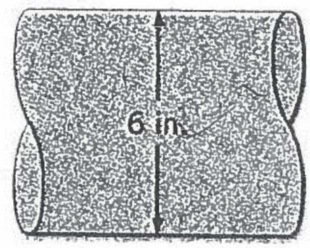
CHEE 314 Fluid Dynamics

Quiz 2 Fall 2010

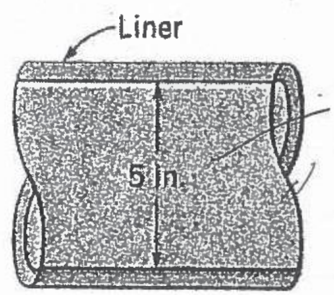
10

1. **(10 marks)** Water flows at a rate of 2.0 ft^3/s in an old rusty 6-in diameter pipe which has a relative surface roughness (ε/D) of 0.010. It is suggested that by inserting a smooth plastic liner ($\varepsilon/D=0$ or smooth) with a diameter of 5-in into the old pipe, the pressure drop per mile (5280 ft) can be reduced. Can the lined 5-in diameter pipe carry 2.0 ft^3/s of water at a reduced pressure drop? Support your answer with appropriate calculations and make all necessary assumptions.

1 mile



Old



New

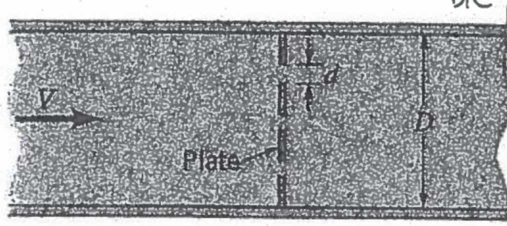
8/10

2. **(10 marks)** A thin flat plate containing a series of holes is to be placed in a pipe to filter out any particles in the liquid. There is some concern that a large pressure drop may develop across the plate and it is proposed to study this problem with a geometrically similar model. It has been determined that the pressure drop (ΔP) is a function of the variables given in the table.
   - 6 a. Use dimensional analysis to develop a suitable set of dimensionless parameters for this problem. 3 parameters
   - 2 b. Determine the appropriate model hole diameter (d) and velocities (V).
   - 2 c. What will be the ratio of pressure drop in the model to prototype ($\Delta Pm/\Delta P$)? One $\boxed{\frac{\Delta P_m}{\Delta P_p} = 1}$

B/C $\rho_m = \rho_p$, $\mu_m = \mu_p$

Prototype	Model
d hole diameter = 1.0 mm	d=? 0.2 mm
D pipe diameter = 50 mm	D=10 mm
μ viscosity = 0.002 $N \cdot s/m^2$	μ = 0.002 $N \cdot s/m^2$
ρ density = 1000 kg/m^3	ρ =1000 kg/m^3
V velocity = 0.1 m/s to 2 m/s	V=? to ?

0.5 m/s → 2 m/s



$\pi_1 = \rho^a \mu^b D^c \Delta P$

$M^0 L^0 T^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{M}{LT}\right)^b L^c \; \frac{M}{LT^2}$

$a+b+1=0$ M $a-2+1=0$ $a=1$

$-b-2=0$ T $b=-2$

$-3a-b+c-1=0$ L

$-3+2+c+1 = -2$

Question 1

$2.54\times10^{-2}\ m = 1\ in$
$30.48\times10^{-2}\ m = 1\ ft$

$Q = 2\ \frac{ft^3}{s}$

$5280\ ft = 1852\ m$ (Back cover of text

$\therefore\ 1\ ft = \frac{1852}{5280}\ m$

$.3048 m$

$\therefore\ 1\ ft = \cancel{0.3507575}\ m$

$Q = 2\times(0.3507575)^3\ \frac{m^3}{s}$

so $1 ft^3 = (0.3507575)^3\ m^3$

$Q = 0.08631\ \frac{m^3}{s}$

$(Q = 0.0567\ m^3/s)$

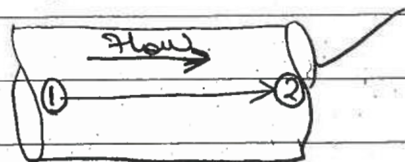
$1\ inch = 0.0254 m$

$\therefore\ D_{old\ pipe} = 6\ in \times \frac{0.0254 m}{in} = 0.1524\ m$

$V_{old\ pipe} = Q/A = \frac{0.08631}{(0.0762)^2\pi} = 4.73\ m/s$ $(3.11 m/s)$

$D_{new\ liner} = 5\ in \times 0.0254\ \frac{m}{in} = 0.127\ m$

ε/D rusty pipe $= 0.010$



old Pipe

Bernoulli's along ① → ②

Assumptions
→ Water @ 20°C
→ Steady State
→ Incompressible liquid
→ $\alpha_i = 1$
→ No shaft work along pipe; $H_S = 0$

By Cons. of Mass, Because pipe' diameter does not change, $V_1 = V_2$
Since $A_1 = A_2$

Assume: flat perfectly horizontal pipe $Z_1 = Z_2$

$$\frac{P_2}{\gamma} + \cancel{\frac{V_2^2}{2g}} + \cancel{Z_2} = \frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + \cancel{Z_1} - \cancel{H_S}^{0} - H_L$$

So we have $H_L = \frac{P_1 - P_2}{\gamma}$ ($P_1 > P_2$ for flow in drawn direction)

$H_{L_{major}} = f\ \frac{l}{D}\ \frac{V^2}{2g}$ So $\frac{f\gamma V^2}{2gD} = \frac{\Delta P}{l}$ } Pressure drop per length.

$Re_{old\ pipe} = \frac{\bar{V} D_{old\ pipe}}{\nu} = \frac{(4.73)(0.1524)}{1.01\times10^{-6}} = 713945 \gg 4$ (469,015)

Flow is turbulent $\therefore\ \frac{1}{\sqrt{f}} = -2\log\left(\frac{2.51}{713945\sqrt{f}} + 0.002703\right)$

Thus

Old pipe $\dfrac{\Delta P}{l} = \dfrac{f \gamma V_{old\,pipe}^2}{2g D_{old\,pipe}} = \dfrac{f \rho \cancel{g} V_{old\,pipe}^2}{2\cancel{g} D_{old\,pipe}}$

$$\frac{\Delta P}{l} = \frac{(0.03799)(998)(4.73^2)}{2(0.1524)}$$

$$\frac{\Delta P}{l} = 2783 \ Pa/m \ \Big\} \text{old pipe}$$

$$V_{new\,pipe} = \frac{Q}{A_{new\,pipe}} = \frac{0.08631}{(0.0635)^2 \pi} = 6.81 \ m/s$$

$$Re_{new\,pipe} = \frac{V D_{new\,pipe}}{\nu} = \frac{(6.81)(0.127)}{1.01 \times 10^{-6}}$$

ε/D for liner $= 0$ $\qquad = 856307 \gg 4200$

Flow is turbulent

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{2.51}{856307\sqrt{f}} + 0 \right)$$

$$\therefore \quad f = 0.01196$$

For new pipe $\dfrac{\Delta P}{l} = \dfrac{f \rho V_{new\,pipe}^2}{2 D_{new\,pipe}} = \dfrac{(0.01196)(998)(6.81^2)}{2(0.127)}$

$$\frac{\Delta P}{l} = 2179 \ \frac{Pa}{m} \ \Big\} \text{new lined pipe}$$

Yes, the lined 5-in diameter pipe can carry 2.0 ft³/s of water at a reduced pressure drop since $\left(\frac{\Delta P}{l}\right)_{lined\ pipe} < \left(\frac{\Delta P}{l}\right)_{old,\ non\text{-}lined\ pipe}$

10

Quiz 2 2010

2)

Variables	Units	Dimensions
ΔP	$kg/m \cdot s^2$	M/LT^2
d	m	L
D	m	L
μ	$kg/m \cdot s$	M/LT
ρ	kg/m^3	M/L^3
V	m/s	L/T

$n = 6 \quad m = 3 \quad \rightarrow \quad 3\ \pi$ terms

repeat ρ, μ, d

$$\Pi_1 = \Delta P \rho^a \mu^b d^c = \left(\frac{M}{LT^2}\right)\left(\frac{M}{L^3}\right)^a \left(\frac{M}{LT}\right)^b (L)^c$$

$M: \ 1 + a + b = 0$

$L: \ -1 - 3a - b + c = 0$

$T: \ -2 - b = 0 \rightarrow b = -2$

$a = 1 \qquad c = 2$

$$\boxed{\Pi_1 = \frac{\Delta P \rho d^2}{\mu^2}}$$

check $\dfrac{(M/LT^2)(M/L^3)L^2}{(M/LT)^2}$ ✓

$$\Pi_2 = D \rho^a \mu^b d^c = L \left(\frac{M}{L^3}\right)^a \left(\frac{M}{LT}\right)^b (L)^c$$

$M: \ a + b = 0$

$L: \ 1 - 3a - b + c = 0$

$T: \ -3a - b = 0$

$a = b = 0 \qquad c = -1$

$$\boxed{\Pi_2 = \frac{D}{d}}$$

$$\Pi_3 = v \rho^a \mu^b d^c = \left(\frac{L}{T}\right)\left(\frac{M}{L^3}\right)^a \left(\frac{M}{LT}\right)^b (L)^c$$

$M: \ a + b = 0$

$L: \ 1 - 3a - b + c = 0$

$T: \ -1 - b = 0 \rightarrow b = -1$

$a = 1 \qquad c = 1$

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CHEE 314 Fluid Dynamics

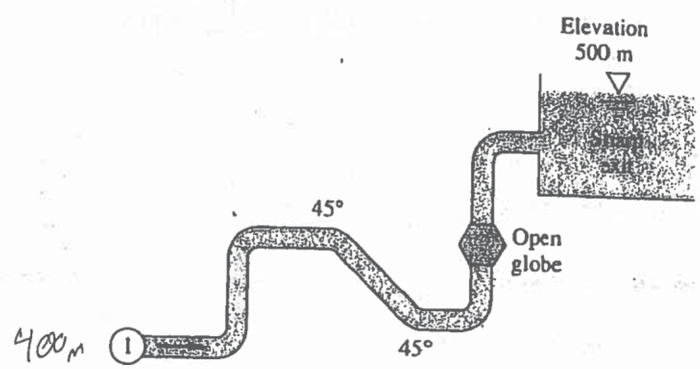
Quiz 2

Fall 2007

1. **(5 marks)** A 1/12 scale model of an airplane is to be tested at 20°C in a pressurized wind tunnel. The prototype is to fly at 240 m/s at 10 km standard altitude. What should be the wind tunnel pressure (in atm) to scale both the Mach number and Reynolds number accurately? Use the data in the table in your calculations. The speed of sound in air at 20°C is 340 m/s. (**HINT…** $Re=(\rho VL)/\mu$, $Ma=V/c$, you may assume air is an ideal gas and viscosity and speed of sound are only dependent on temperature).

Elevation	Air Speed of Sound c (m/s)	Pressure (kPa)	Density (kg/m^3)	Viscosity (kg/(ms))
10 km	299	26.5	0.4125	1.47E-5

2. **(10 marks)** The piping system shown in the figure consists of 1200 m of 5 cm diameter cast iron pipe. If the elevation at point 1 is 400m, what gage pressure is required at point 1 to deliver 0.005 m^3/s of water at 20°C into the reservoir? Account for all losses in the system.



Quiz 2 - 2007

1.

Actual

10 km

240 m/s

$\mu = 1.47\times10^{-5}\ \frac{kg}{m\cdot s}$

$\rho = 0.4125\ kg/m^3$

$c = 299\ m/s$

Model

$P_M = ?$

$V_M = ?$

$\rho_M = ?$

$c = 340\ m/s$

$\mu_M = 1.81\times10^{-5}\ \frac{kg}{m\cdot s}$

$$Ma_A = Ma_M$$

$$\frac{V_A}{c_A} = \frac{V_M}{c_M}$$

$$\frac{240}{299} = \frac{V_M}{340} \Longrightarrow V_M = 272.9\ m/s$$

$$Re_A = Re_M$$

$$\frac{(0.4125)(240)L}{1.47\times10^{-5}} = \frac{\rho_M (272.9)\frac{L}{12}}{1.81\times10^{-5}}$$

$$\rho_M = 5.36\ kg/m^3$$

for an ideal gas $P = \rho \frac{R}{M_M} T$ Table A.6

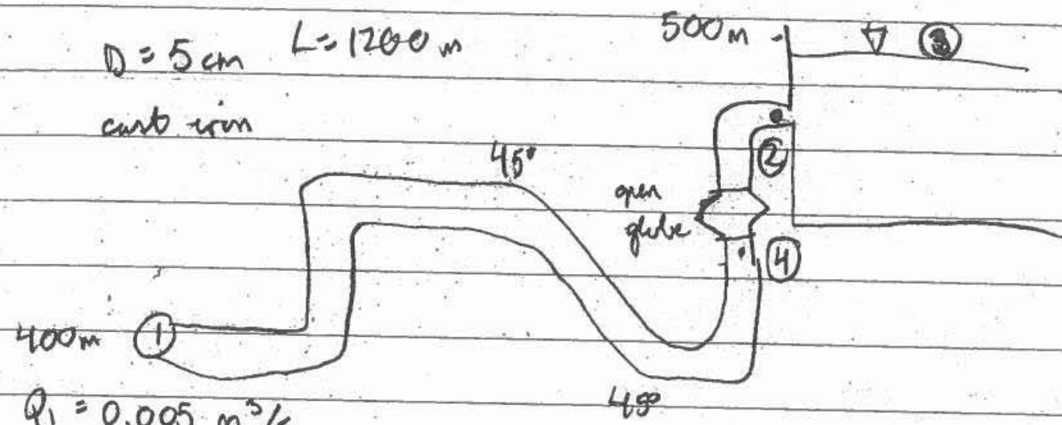
$$P = (5.63\ kg/m^3)(286.9\ \tfrac{J}{kg\cdot K})(293\ K)$$

$$P = 4.7\times10^5\ Pa$$

$$\underline{P = 4.7\ atm}$$

2. $D = 5\text{ cm}$ $L = 1200\text{ m}$

cast iron



$Q_1 = 0.005\ m^3/s$

$T = 20°C$

assume: - large reservoir

- incompressible
- steady flow

using continuity:

$$Q_1 = Q_2$$

$$A_1 = A_2 \qquad \therefore \quad V_1 = V_2$$

$$V_2 = V_1 = Q_1/A_1 = 0.005\,m^3/s \,/\, \pi(0.05\,m/2)^2 = 2.546\ m/s$$

Bernoulli:

$$\underbrace{\cancel{P_3/\gamma}}_{0 \to atm} + \underbrace{\cancel{V_3^2/2g}}_{0 \to \text{large reservoir}} + Z_3 = P_1/\gamma + V_1^2/2g + Z_1 - \underbrace{\cancel{H_S}}_{\text{no work}} - H_{L\,major} - H_{L\,minor}$$

$$P_1 = \gamma\,(Z_3 + H_{L\,major} + H_{L\,minor} - V_1^2/2g - Z_1)$$

$$H_{L\,major} = f\,\frac{L}{D}\,\frac{\bar{V}^2}{2g} \leftarrow \text{need } f \rightarrow \text{is flow laminar or Turbulent}$$

$$Re = \rho \bar{V} D/\mu = \bar{V} D/\nu = (2.546\ m/s)\cdot(0.05\,m) \,/\, 1.01\times10^{-6}\ m^2/s$$

$$\nu @ 20°C = 1.01\times10^{-6}\ m^2/s$$

$$= 126\,040 \quad > 4200 \leftarrow \text{turbulent}$$

for cast iron : $\varepsilon = 0.26$ mm

$\therefore \varepsilon/D = 0.00026\,m / 0.05\,m = 0.0052$

using colebrook:

$f_0 = 0.25 [\log(\varepsilon/D / 3.7 + 5.74 / Re^{0.9})]^{-2}$
$= 0.0317$

$1/f_1^{0.5} = -2.0 \log(\varepsilon/D / 3.7 + 2.51 / Re\, f_0^{0.5})$

$f_1 = 0.0315$

$\therefore H_{L\,major} = 0.0315 \cdot 1200\,m / 0.05\,m \; (2.546\,m/s)^2 / 2 \times 9.81\,m/s^2$

$= 249.8$ m

$H_{L\,minor} =$ ① bends → 4 90° → $Le/D = 30$
2 45° → $Le/D = 16$

$H_{e\,minor①} = 4\,(f \overset{30}{Le/D}\, \bar{V}^2/2g) + 2\,(f \overset{16}{Le/D}\, \bar{V}^2/2g)$
$= 0.0315 \cdot (2.546\,m/s)^2 / 2 \cdot 9.81\,m/s\; (120 + 32)$

~~1.2.0~~

$= 1.58$ m

② globe : $Le/D = 340$

$H_{e\,minor②} = 340 \cdot 0.0315 \cdot (2.546)^2 / 2 \cdot 9.81 = 3.53$ m

③ entrance $K = \cancel{\ }$

$\cancel{H_{L_{minor}} = 0.5 \cdot (2.546)^2 / 2 \times 9.81}$

$= 0.$ $\cancel{K = 0}$ $\rightarrow$ $A_1/A_2 \approx 0$ $\uparrow$ large reservoir

$K = 1$

$H_{L_{minor} ③} = 1 \cdot (2.546\ m/s)^2 / 2 \cdot 9.81\ m/s^2 = 0.33\ m$

$\therefore\ H_{L_{minor}} = 1.58 + 3.53 \overset{+0.33}{\wedge} = \cancel{5.11\ m}$ $5.44\ m$

$H_{L_{total}} = \cancel{5.11}\ 5.44 + 249.8 = \boxed{255\ m}$

$P_1 = 9.81\ m/s \cdot 998\ kg/m^3\ (500m - 400m + 255m - (2.546\ m/s)^2 / 2.$

$= \cancel{-1520\ 743}\ N/m^2$ ← makes no sense

$= \cancel{-1520}$

$= 3\,472\ kPa$

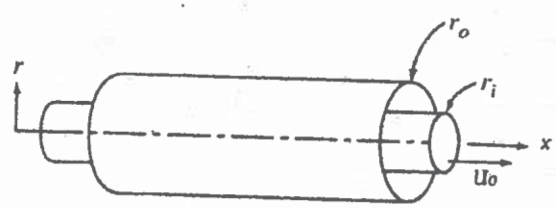
10

Department of Chemical Engineering
McGill University

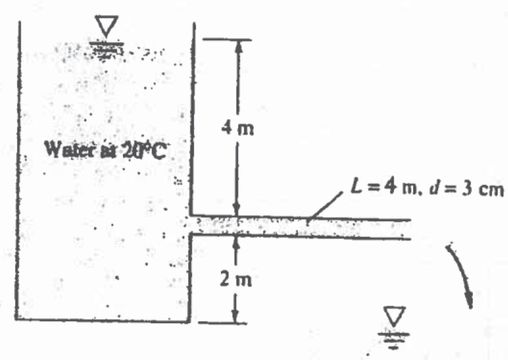
CHEE 314 Fluid Dynamics

Quiz 2 Fall 2006

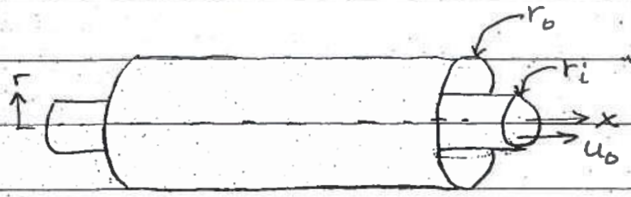
1. Flow is created in a horizontal pipe by pulling a rod through the center at a constant velocity Uo, Figure 1. a) Derive an expression for the fully developed laminar flow between the pipe and the rod. What is the shear stress that acts on b) the pipe wall and c) rod?



2. The tank system shown in the figure is used to deliver at least 11 m^3/hr of water at 20°C to the reservoir through a 3 cm diameter pipe. If the pipe is 4 m long, estimate what the maximum roughness (e) can be to achieve the desired flow rate? Suggest a pipe material.



1



Assume
- No slip @ wall
- Incompressible
- Steady state
- Newtonian

a) $u_\theta = 0$

$u_r = 0$

$u_x(r)$

$\frac{dP}{dz}$ = constant (s.s.) $\quad \frac{dP}{d\theta} = \frac{dP}{dr} = 0$

$r = r_o \quad u_x(r_o) = 0$
$r = r_i \quad u_x(r_i) = u_o$
} Boundary Conds.

Continuity Eqn

$$\frac{1}{r}\frac{\partial}{\partial r}\cancel{(r u_r)}^{0} \; \frac{1}{r}\cancel{\frac{\partial u_\theta}{\partial \theta}}^{0} + \cancel{\frac{\partial u_x}{\partial x}}^{0} = 0$$ satisfied

Navier-Stokes

$$0 = -\frac{\partial P}{\partial x} + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_x}{\partial r}\right)\right)$$

$$\frac{r}{\mu}\frac{\partial P}{\partial x} = \frac{\partial}{\partial r}\left(r\frac{\partial u_x}{\partial r}\right)$$

$$\int \frac{r}{\mu}\frac{dP}{dx}\,dr = \int \partial\left(r\frac{\partial u_x}{\partial r}\right)$$

$$\frac{r^2}{2\mu}\frac{dP}{dx} + C_1 = r\frac{\partial u_x}{\partial r}$$

$$\int\left[\frac{r}{2\mu}\frac{dP}{dx} + \frac{C_1}{r}\right]dr = \int \partial u_x$$

$$\frac{r^2}{4\mu}\frac{dP}{dx} + C_1\ln(r) + C_2 = u_x$$

BCs

$$u_x(r_o) = 0 = \frac{r_o^2}{4\mu}\frac{dP}{dx} + C_1\ln(r_o) + C_2$$

$$-\frac{r_o^2}{4\mu}\frac{dP}{dx} - C_1\ln(r_o) = C_2$$

$$u_x(r_i) = u_o = \frac{r_i^2}{4\mu}\frac{dP}{dx} + C_1\ln(r_i) - \frac{r_o^2}{4\mu}\frac{dP}{dx} - C_1\ln(r_o$$

$$\frac{u_o - \frac{dP}{dx}\frac{1}{4\mu}\left(r_i^2 - r_o^2\right)}{\ln(r_i/r_o)} = C_1$$

$$u_x(r) = \frac{r^2}{4\mu}\frac{dP}{dx} + \left(\frac{u_0 - \frac{dP}{dx}\frac{1}{4\mu}(r_i^2 - r_0^2)}{\ln(r_i/r_0)}\right)\ln(r) - \frac{r_0^2}{4\mu}\frac{dP}{dx} - \left(\frac{u_0 - \frac{dP}{dx}\frac{1}{4\mu}(r_i^2 - r_0^2)}{\ln(r_i/r_0)}\right)\ln(r$$

b) $\tau = \mu \frac{du_x}{dr}$

$$\tau = \frac{r}{2}\frac{dP}{dx} + \mu\left[\frac{u_0 - \frac{dP}{dx}\frac{1}{4\mu}(r_i^2 - r_0^2)}{\ln(r_i/r_0)}\right]\frac{1}{r}$$

on pipe wall $r = r_0$

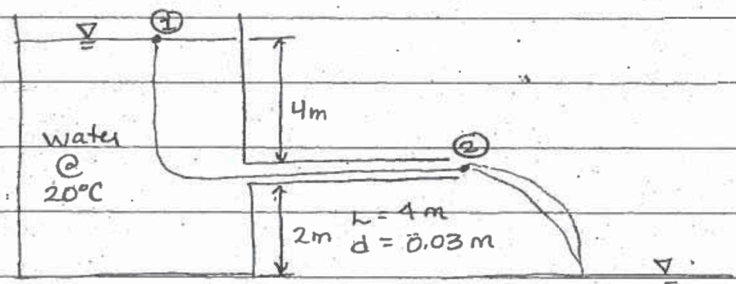
$$\tau = \frac{r_0}{2}\frac{dP}{dx} + \left(\frac{u_0\mu - \frac{dP}{dx}\frac{1}{4}(r_i^2 - r_0^2)}{\ln(r_i/r_0)}\right)\frac{1}{r_0}$$

On rod $r = r_i$

$$\tau = \frac{r_i}{2}\frac{dP}{dx} + \left(\frac{u_0\mu - \frac{dP}{dx}\frac{1}{4}(r_i^2 - r_0^2)}{\ln(r_i/r_0)}\right)\frac{1}{r_i}$$

10

2



Assume
- Incompressible
- ~~Inviscid~~
- Uniform flow ($\alpha = 1$)
- Reservoir is BIG
- No shaft work

$$Q = 11\ \frac{m^3}{hr} \times \frac{1\ hr}{60\ min} \times \frac{1\ min}{60\ s} = 3.056 \times 10^{-3}\ \frac{m^3}{s}$$

$$\frac{Q}{A} = \bar{V}$$

$$\bar{V} = \frac{3.056 \times 10^{-3}}{\frac{\pi}{4}(0.03)^2} = 4.323\ m/s$$

Bernoulli's Eqn

$$\cancelto{0\ atm}{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + z_2 = \cancelto{0\ atm}{\frac{P_1}{\gamma}} + \cancelto{0}{\frac{V_1^2}{2g}} + z_1 - \cancelto{0}{h_s} - h_{L_{min}} - h_{L_{maj}}$$

$$h_L = h_{L_{maj}} + h_{L_{min}}$$

$$Re = \frac{\bar{u} D}{\nu} = \frac{4.323\,(0.03)}{1.01 \times 10^{-6}} = 128,400$$

$$h_{L_{min}} = 0.5\,\frac{\bar{V}^2}{2g}$$

$$h_{L_{maj}} = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$$

$$h_{L_{min}} = \frac{0.5\,(4.323)^2}{2\,(9.8)}$$

$$\frac{V_2^2}{2g} + (z_2 - z_1) + h_{L_{min}} = -h_{L_{maj}}$$

$$h_{L_{min}} = \cancel{0}.01103$$

$$\frac{(4.323)^2}{2(9.8)} + (-4.015) + 0.01103 = -f\left(\frac{4}{0.03}\right)\left(\frac{4.323^2}{2(9.8)}\right)$$

$$-0.024 = -f$$

$$0.024 = f$$

$$\boxed{e = 0.0517\ mm}$$

Colebrook Eqn

$$\frac{1}{f^{0.5}} = -2.0 \log\left(\frac{e/D}{3.7} + \frac{2.51}{Re\, f^{0.5}}\right)$$

Suggested pipe material: Commercial Steel

$$\frac{1}{0.024^{0.5}} = -2 \log\left(\frac{e}{0.111} + 1.262 \times 10^{-4}\right)$$

$$5.923 \times 10^{-4} = \frac{e}{0.111} + 1.262 \times 10^{-4}$$

B

5.17×10^{-5}

McGill University
Faculty of Engineering

FINAL EXAMINATION
FALL 2012 (DECEMBER '12)

Student Name:		McGill ID:									

Fluid Mechanics
CHEE 314, Fall 2012

Examiner: Richard L. Leask

Co-Examiner: Phillip Servio

Signature: ____________________

Signature: ____________________

Date: Friday December 7, 2012

Time: 14:00 hrs

OPEN BOOK EXAM: No restriction on calculators

ALLOWED MATERIAL:

1. No restrictions on Texts, Notes or Calculators
2. No solution manuals, or CHEE 314 old tests or course packs

OTHER INSTRUCTIONS:

Answer all **6 questions** in the provided examination booklet and clearly indicate the number of booklets used. Good Luck and Happy Holidays.

1a. [5 Marks] Short Answer:

(a) Name one model of a non-Newtonian fluid.

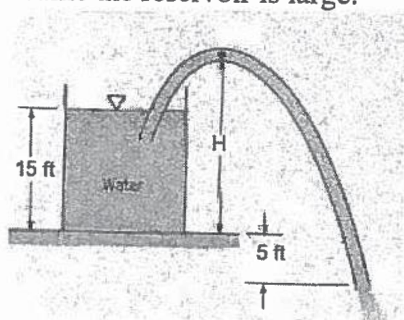
(b) Why does the overall drag reduce on a sphere when the flow becomes turbulent?

(c) What is the Magnus effect?

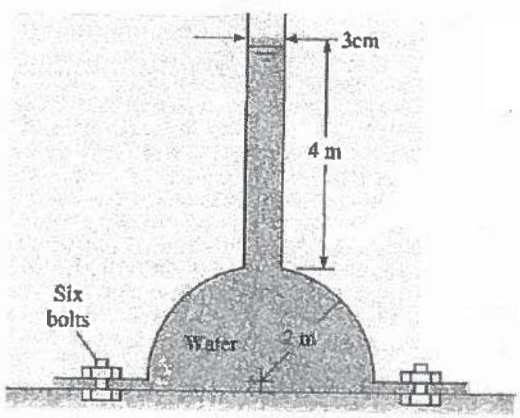
(d) When does separation in a flow field occur?

(e) True of False (and provide a brief explanation): In open channel flow, the pressure across straight streamlines is constant.

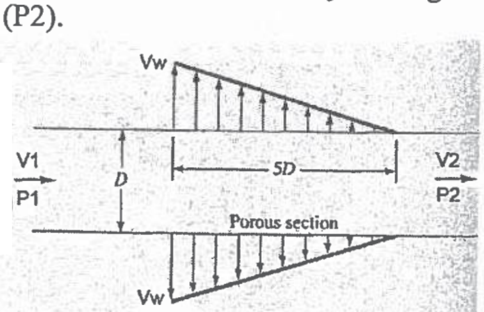
1b. [5 marks] Cavitation can limit the ability to siphon a fluid from a tank by causing gas bubbles to form when the pressure is reduced below the vapor pressure of the fluid. Given the setup in the figure, what is the maximum height H the siphon hose can reach without cavitation occurring. You may assume the water is at 60°F and the exit is to atmosphere (14.7 psia). You may neglect head losses in the system and assume the reservoir is large.



2. [6 marks] A 2-m radius hemispherical dome weights 30 kN and is filled with water. It is anchored to the floor by 6 equally spaced bolts. What is the force on **each** bolt required to hold down the dome?

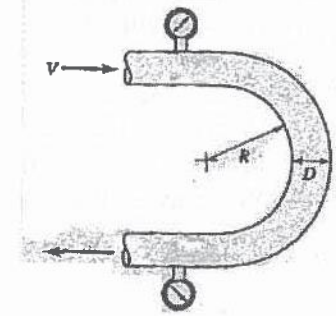


3. [10 marks] In a pipe manifold, fluid is removed from a porous wall section of the pipe (5D long). Assume the flow is incompressible and neglect viscous losses. If the velocity of the fluid leaving at the start of the porous section is Vw and zero at the end (ie. Linear reduction along the porous section), find **A)** an expression for the velocity leaving the pipe (V2) and **B)** an expression for the pressure at the exit (P2).

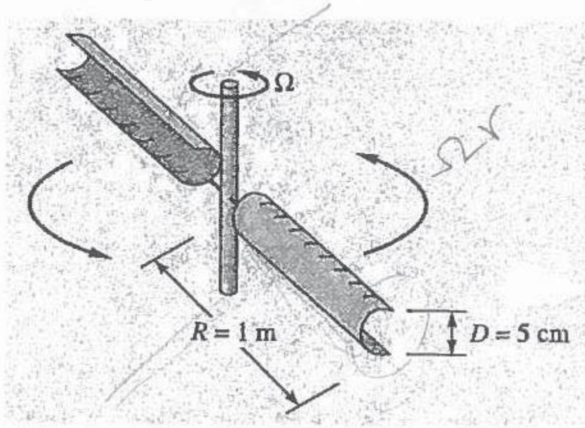


4. **[10 marks]** A fluid flows through a horizontal curved pipe with a velocity V. The pressure drop, ΔP, between the entrance and the exit of the bend is thought to be a function of the velocity (V), the bend radius (R), pipe diameter (D) and fluid density (ρ). **A)** Perform dimensional analysis to come up with a set of non-dimensional terms. The data shown were collected in the laboratory for a fluid with a density of ρ=2.0 slugs/ft^3, R=0.5 ft and D=0.1 ft. **B)** Use this data to test your set of non-dimensional terms. Were the variables used correct?

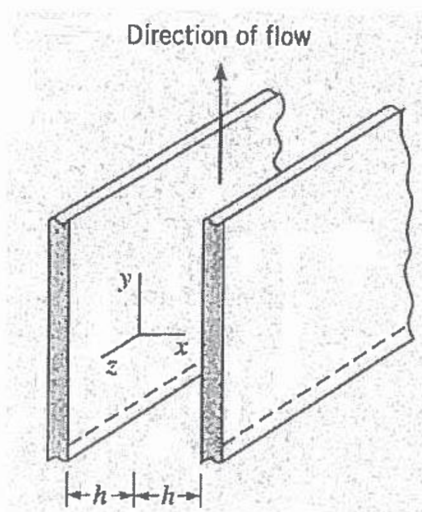
V(ft/s)	2.1	3.0	3.9	5.1
ΔP	1.2	1.8	6.0	6.5



5. **[8 marks]** A rotary mixer consists of two 1-m-long half-tubes rotating around a central arm as shown in the figure. If the fluid is water at 20°C and the maximum driving power available is 20 kW, what is the maximum rotation speed?



6. **[12 marks]** A viscous incompressible fluid flows upward between two parallel plates. An applied constant pressure gradient is used to overcome the effect of gravity. Use the axis given (origin in the center of the channel) and solve for **A)** the velocity profile between the plates and **B)** the average velocity. You may assume the plates are infinitely wide into the page (ie. Z direction) and the flow is laminar.

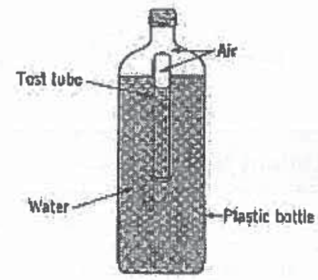


1a. [5 Marks] Short Answer:

- **(a)** Give an example of a Bingham Plastic Fluid. – Toothpaste, ketchup
- **(b)** What causes a meniscus to form? – surface tension
- **(c)** When does flow through a pipe become fully turbulent? $Re > 4300$
- **(d)** How can a perfect sphere create lift? spin
- **(e)** What determines the length of the entrance length (Le) in laminar flow? Re

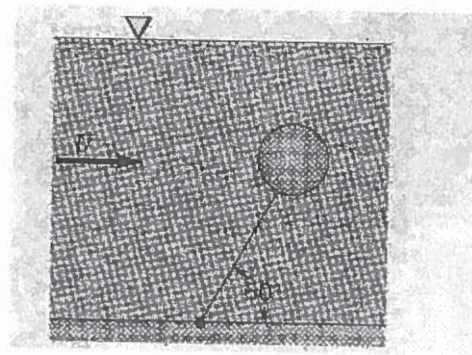
5

1b. [~~8~~ marks] An inverted test tube that is partially filled with air floats in a plastic water-filled pop bottle as shown in the figure. The amount of air is adjusted so that it just floats. The cap is then placed on the bottle. If the bottle is squeezed, the test tube will sink to the bottom. Explain this phenomenon.



$$\left.\begin{array}{l}\text{– Pressure increases} \\ \text{– compressed air} \\ \text{– Boyancy reduced}\end{array}\right\} 5$$

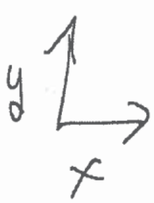
2. **[10 marks]** A 2-in-diameter cork sphere (specific weight 13 lb/ft^3) is attached to the bottom of a river with a thin cable. If the sphere has a drag coefficient of Cd=0.5, determine the river velocity U when the cable angle is 60° (see figure). You may assume the drag on the cable and the weight of the cable are negligible. Assume the river has a temperature of 60°F.

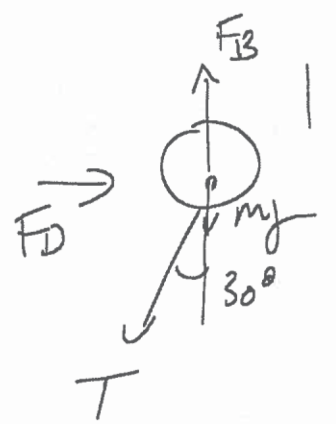


Assume

- Steady flow 1
- incomp
- neglect cable

FBD





$$F_D = C_D \tfrac{1}{2} \rho V^2 A \quad /$$

$$F_B = \gamma_w \forall \quad /$$
$$= 62.4 \forall$$

$$\forall = \tfrac{4}{3}\pi \left(\tfrac{1}{12}\right)^3$$
$$= 0.0024 \text{ ft}^3$$

$$\Sigma F_y = 0$$

$$F_B = T\cos 30° + mg \quad 2$$

$$T = \frac{62.4\forall - mg}{\cos 30°} = \frac{\rho g \forall - \gamma_c \forall}{\cos 30°}$$

$$= \frac{(62.4 - 13)\,0.0024}{\cos 30°} = 0.138 \text{ lbf} \quad /$$
$$0.614 \text{ N}$$

$\therefore \quad \Sigma F_x = 0$

$$F_D = T \sin 30^\circ \quad 2$$

$$0.5 \; \tfrac{1}{2} \rho A V^2 = 0.138 \sin 30^\circ$$

$$\tfrac{1}{4} \rho \pi \left(\tfrac{1}{12}\right)^2 V^2 = 0.138 \cdot \sin 30^\circ$$

$$V = 2.561 \text{ ft/s}$$

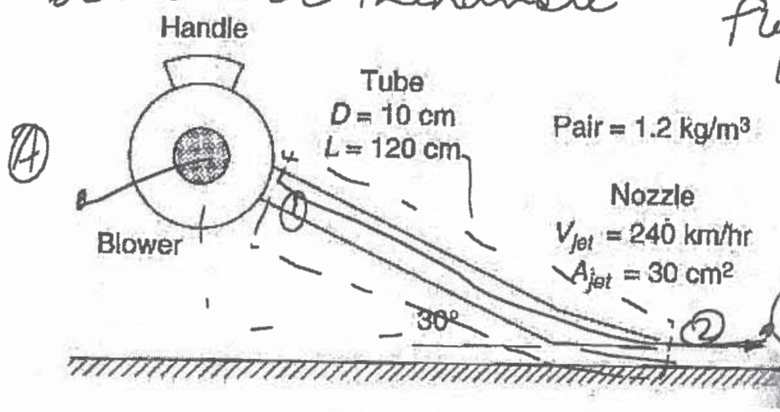
$$0.78 \text{ m/s}$$

20°C

A sudden contraction

3. [~~14~~ 12 marks] Air enters the centrifugal air-pump of a leaf blower in the center (shaded area). The air flows out though a 10-cm diameter smooth tube of length 120 cm. ~~A nozzle~~, with an exit area of 30-cm², is attached to produce an exit velocity of 240 km/hr.
   a. What is the flow rate produced by the pump?
   b. If the pump is 65% efficient, what is the required horsepower of the pump?
   c. What ~~horizontal~~ force must be applied to the handle located 30 cm above the nozzle? from the flow leaving the blower?

moment will be created @ the handle



Assume
- steady
- incomp

C of Mass

$$Q_1 = Q_2$$

$$V_1 A_1 = V_2 A_2$$

$$V_1 = V_2 A_2 / A_1$$

(A) → $= 25.5\ m/s$

$A_1 = \pi (0.05)^2 = 0.0079\ m^2$

$A_2 = 30\ \frac{cm^2}{} \left| \frac{m^2}{(100\ cm)^2} \right.$

$= 0.003\ m^2$

$V_2 = \frac{240\ km}{hr} \left| \frac{1\ hr}{60 \times 60} \right| \frac{1000\ m}{km}$

$= 66.7\ m/s$

$A_2 / A_1 = 0.382$

(B) Bernoulli's (A) → (B)

$$\cancel{\frac{P_B}{\gamma}} + \cancel{\frac{V_B^2}{2g}}^{0} + \cancel{Z_B} = \cancel{\frac{P_A}{\gamma}} + \frac{V_A^2}{2g} + \cancel{Z_a} - H_L - H_S$$

$$\frac{V_B^2}{2g} = -H_L - H_S \quad 2$$

major $\qquad$ contraction $\cdot K \approx 0.3$

$$H_L = f\frac{L}{D}\frac{V_1^2}{2g} + K\frac{V_2^2}{2g} \quad 1$$

$$Re = \frac{V_1\, 0.1}{1.5\times10^{-5}} = 170\,000 \quad \text{Turbulent}$$

$\epsilon/D \rightarrow$ smooth $\cdot\ f \cong 0.016$

$$\therefore H_L = 0.016\,\frac{1.2}{0.1}\,\frac{V_1^2}{2(9.81)} + 0.3\,\frac{V_2^2}{2(9.81)}$$

$$= 74.3\,m \quad 1$$

$$\frac{V_2^2}{2g} = -74.3 - H_s$$

$$-H_s = 300.93 \quad m$$

$$\text{Power} = \gamma H_s Q$$

$$= 1.21(9.81)(300.89)(V_2 A_2)$$

$$= 712.2 \text{ Watts}$$

$$\frac{746 W}{HP} = 0.95 \text{ hp}$$

@ 65% efficiency $1.47 hp$

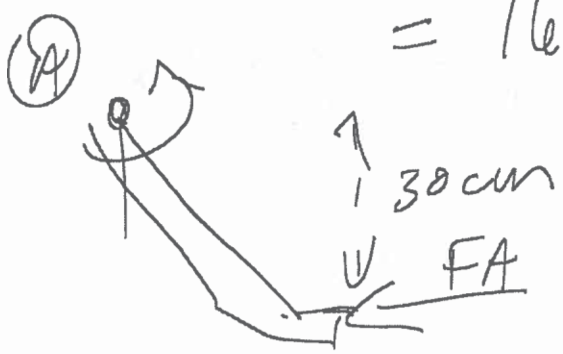
C of Momentum.

$$\frac{d}{dt}\int \rho \,dV + \int_{cs} \vec{V}\rho\vec{V}\cdot\hat{n}\,dA = \Sigma F_A$$

$$F_A = V_2 \rho V_2 A$$

$$= V_2^2 (1.21) A_2$$

$$= 16.13\,N$$



should be 0.6? (120 cm) sin 30°
Yep.

$$M_a = F_a \times 0.3$$

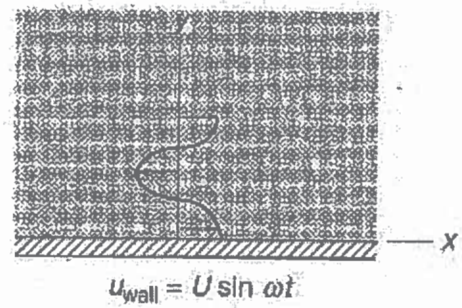
$$= 4.8\ N\cdot m$$

$M_0 = 9.6\ N\cdot m$

$M_0 = 9.8\ J$

8

4. **[10 marks]** A large flat plate oscillates beneath a liquid as shown in the figure.
   a. Prove the velocity u cannot be a function of x if the flow is only parallel to the plate.
   b. Derive a differential equation that describes the motion of the fluid if the flow is only parallel to the plate. **DO NOT** try to solve the equation.
   c. What boundary conditions would you use if you did solve the equation?



Assume 2

- Newtonian
- incomp
- no slip
- laminar

$u(y)$, $\frac{\partial p}{\partial x} = 0$ 1

$V = W = 0$

Continuity

$$\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}} + \cancel{\frac{\partial w}{\partial z}} = 0$$

u is not a fnc of x

1

N-S/

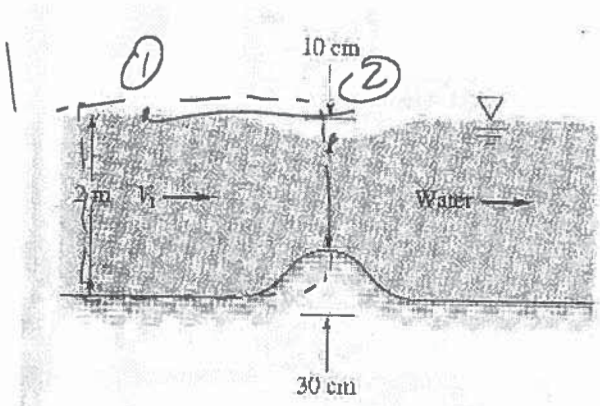
x/ $$\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial y} \quad 2$$

$$u(0) = U \sin\omega t \quad 1$$

$$u(\infty) = 0 \quad 1$$

5. [~~10~~ 8 marks] As a water (20°C) passes ~~of~~ over a hump on the bottom of a river, a dip (Δh) in the water level can be observed and can serve as a measurement of flow (Q). If Δh = 10-cm when the bump is 30-cm high, what is the volume flow Q. You may assume there are no losses and the width of the river, width=b, is large. Leave your answer in terms of b.

Assume
- steady
- incomp



C of Mass

$$V_1 A_1 = V_2 A_2$$

$$V_1 (2 \times \cancel{b}) = V_2 (2 - 0.4) \cancel{b}$$

$$V_2 = 1.25 V_1 \qquad 2$$

Bernoulli's 1 → 2

$$\frac{\cancel{P_2}}{\cancel{\gamma}} + \frac{V_2^2}{2g} + z_2 = \frac{\cancel{P_1}}{\cancel{\gamma}} + \frac{V_1^2}{2g} + z_1$$

$$(1.25V_1)^2 = V_1^2 + 2g(0.1)$$

$$V_1^2 = \frac{2(9.81)(0.1)}{(1.25^2-1)}$$

$$V_1 = \sqrt{\frac{2(9.81)(0.1)}{1.25^2-1}} \quad 2$$

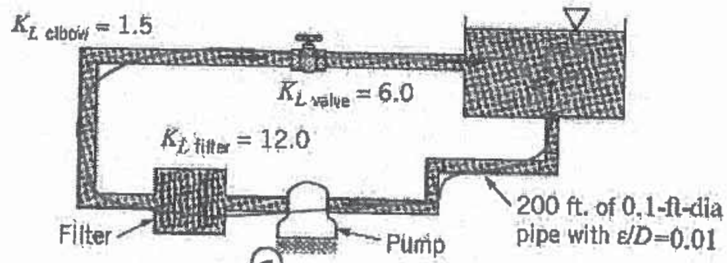
$$= 1.87 \text{ m/s}$$

$$\left.\begin{aligned} &V_2 = 2.33 \text{ m/s} \\ &Q = 3.74 \cdot b \text{ m}^3\text{/s} \end{aligned}\right\} 2$$

6. **[1~~0~~2 marks]** Water (~~5°C~~ 40°F) is circulated from a large tank, through a filter and back to the tank as shown in the figure. The power added to the water by the pump is 200 ft·lb/s. Determine the flow rate through the filter. The total length of pipe is 200-ft with a diameter of 0.1-ft. The relative roughness ε/D =0.01. Assume all the elbows have a loss coefficient K_L=1.5. The valve has a loss coefficient K_L=6.0 and the filter K_L=12.0. Account for all minor losses.

Assume
- steady
- incomp

1



200 lb ft/s
Power

Bernoulli's ① → ①

$P = \gamma H_s Q$

1

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 - H_L - H_s$$

$$H_L = H_s$$

$$\gamma = 1.94(32.2) = 62.4\ lb/ft^3$$

$$A = \pi(0.05)^2 = 0.0079\ ft^3$$

$$H_L = \frac{200}{62.4\,VA} = \frac{408.09}{V}$$

1

H_L

Major $H_L = f \frac{L}{D} \frac{V^2}{2g}$

minor $H_L = \frac{V^2}{2g}\left(5 \times 1.5 + \overset{2}{12} + 0.5 + 1 + 6\right)$

entrance expansion

valve

$= \frac{27V^2}{2g}$

$\therefore \quad \frac{408.09}{V} = f \frac{200}{0.1} \frac{V^2}{2g} + \frac{27V^2}{2g}$

$\epsilon/D = 0.01 \quad$ assume $f = 0.036 \;\; 2$

$\frac{2.63 \times 10^4}{V} = 2000f V^2 + 27 V^2$

$V = \sqrt[3]{\frac{2.63 \times 10^4}{2000f + 27}}$

$$V = 6.38\ ft/s$$

$$\therefore Re = \frac{VD}{\nu} = \frac{6.38\,(0.1)}{1.65\times10^{-5}} \quad 2$$

$$= 3.86\times10^{5}$$

Colebrook $\quad f = 0.0355$

$$\therefore V = 6.32\ ft/s$$

$$\therefore Q = 0.05\ ft^3/s \quad 2$$

McGill University
Faculty of Engineering

FINAL EXAMINATION
FALL 2006 (DECEMBER '06)

STUDENT NAME

MCGILL I.D. NUMBER

Fluid Mechanics
CHEE 314, Fall 2006

Examiner: Richard L. Leask

Signature:

Co-Examiner: Milan Maric

Signature: Milan Maric

Date: Monday December 18, 2006

Time: 9:00 hrs

CLOSE BOOK EXAM: No restriction on calculators

ALLOWED MATERIAL:

1. No restrictions on Texts, Notes or Calculators

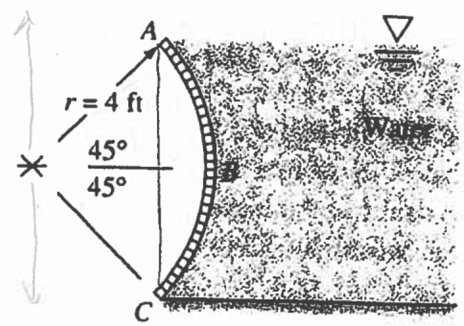
OTHER INSTRUCTIONS:

Answer all **6 questions** in the provided examination booklet and clearly indicate the number of booklets used. Good Luck and Happy Holidays.

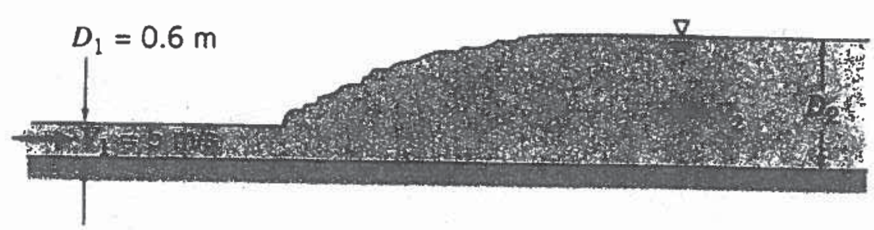
1a. [5 Marks] Short Answer:
- **(a)** What is the pressure gradient across straight streamlines?
- **(b)** Give two assumptions used when estimating the velocity with a Pitot-static tube.
- **(c)** What is the shear stress at a free surface?
- **(d)** Fully developed turbulent pipe flow causes greater wall shear stress than fully developed laminar pipe flow (**True or False**).
- **(e)** In smooth pipes, the friction factor can be reduced by working at high Reynolds numbers (**True or False**).

1b. [5 marks] The speed of propagation (C – units m/s) of a capillary wave is known to be only a function of the density (ρ), wavelength (λ – units m) and surface tension (Y – units N/m). For a given density and wavelength, how does the propagation speed change if the surface tension is doubled?

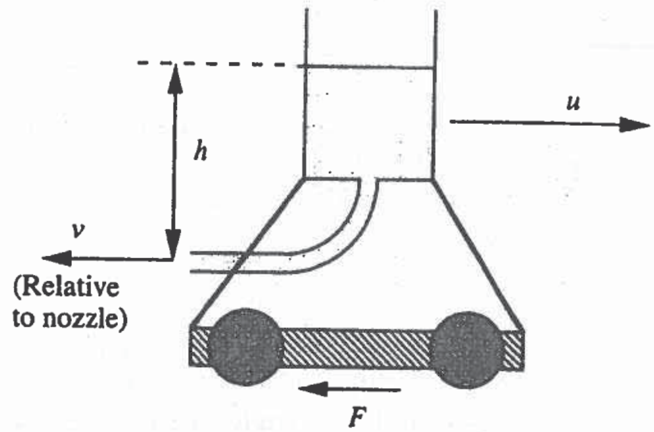
2. [10 marks] The gate ABC in the figure is a quarter circle or radius 4ft and is 8 ft wide into the page. Compute the horizontal and vertical hydrostatic forces on the gate and the line of action of the resultant force (angle and location). Assume the water is at 60°F.



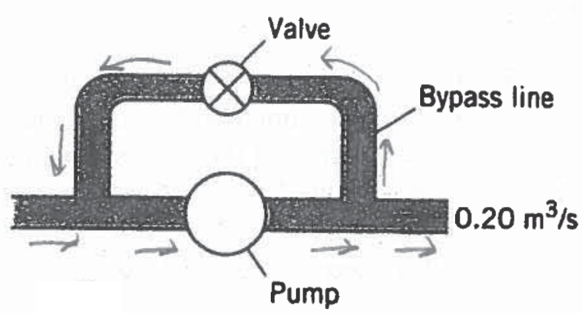
3. [10 marks] A sudden change in depth (a hydraulic jump) can occur in a wide horizontal open channel flow under some conditions. Consider a channel of width *w* with an incoming average velocity of 5 m/s and depth of 0.6 m as shown in the figure. What is the depth and velocity (D_2 and V_2) at the exit of the channel? You may neglect friction losses.



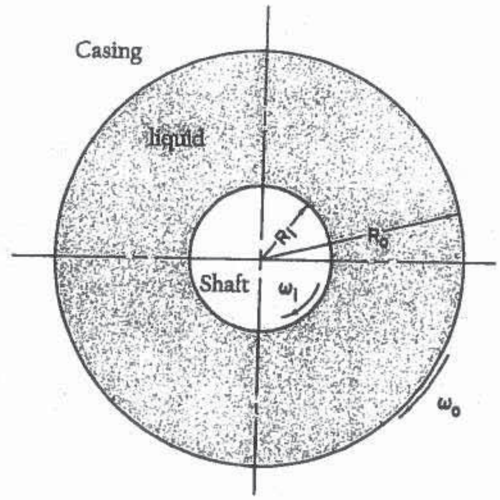
4. **[10 marks]** A fluid powered cart is shown in the figure. Neglect viscosity and you may assume the reservoir is large.
   a. Use Bernoulli's equation to derive an expression for the velocity (v) **relative to the nozzle** in terms of the current height differential (h).
   b. Derive an expression for the acceleration of the cart in terms of any or all the variables h, A (area of the nozzle), F (the drag force), u (cart velocity), M (current mass of the cart), ρ (fluid density) and any other physical constants you think are necessary.



5. **[10 marks]** A bypass line is used to recirculate fluid as shown in the figure. The bypass valve is used to control the flow rate in the system. The head supplied by the pump is given by $H_p=100-100Q$, where H_p is in meters and Q is in m^3/s. The bypass line is 10 cm in diameter. Assume the only head loss is that due to the valve, which has a head-loss coefficient k=0.2. The discharge leaving the system is 0.2 m^3/s. Find the flow rate through the pump and the bypass line. (HINT, H_p can also be estimated by the pressure drop across the pump, $H_p=\Delta P/\gamma$)



6. **[10 marks]** A viscous liquid of constant ρ and μ is used to lubricate a shaft in a pipe casing. The flow is fully developed and there is no applied pressure gradient. The inner shaft of radius R_i rotates at an angular rate of ω_i. The outer casing (a pipe) of radius R_o also rotates at an angular rate of ω_o. Derive an expression of the velocity profile and flow rate between the shaft and casing. The length of each component into the page is b.



2006 Final Questions 1b), 5.

1b) $C = f(\rho, \lambda, Y)$

n: dimensional parameters $(C, \rho, \lambda, Y) = 4$

$[C] = \frac{L}{t}$ $\quad [\rho] = \frac{m}{L^3}$ $\quad [\lambda] = L$ $\quad [Y] = \frac{m}{t^2}$

r: primary dimensions $(L, m, t) = 3$

$m = r = 3$ repeating parameters (ρ, λ, Y)

$n - m = 4 - 3 = 1$

$\pi_1 = C \rho^a \lambda^b Y^c$

$L: 0 = 1 - 3a + 1b + 0c$

$t: 0 = -1 + 0a + 0b - 2c \Rightarrow c = -\frac{1}{2}$

$m: 0 = 0 + 1a + 0b + 1c \Rightarrow a = -c =$

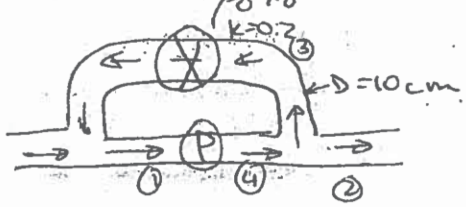
$b = \frac{1}{2}$

$\Pi_1 = C\rho^{\frac{1}{2}} \lambda^{\frac{1}{2}} Y^{-\frac{1}{2}} = C\sqrt{\frac{\lambda\rho}{Y_1}} = \text{constant}$

$C = \text{constant}\sqrt{\frac{Y}{\lambda\rho}}$ $\qquad Y = 2Y_0 \quad C = ?$

$C = cst\sqrt{\frac{2Y_0}{\rho_0 \lambda_0}} = C_0\sqrt{2} \Rightarrow \underline{C = \sqrt{2}\, C_0}$

5.



Assumptions

1) steady
2) incompressible
3) 1-D
4) neglect gravity

$H_p = 100 - 100Q$

$Q_1 = Q_2 + Q_3$

Bernoulli's

$$\cancel{\frac{P_4}{\gamma}} + \cancel{\frac{V_4^2}{2g}} + \cancel{z_4} = \cancel{\frac{P_4}{\gamma}}\ \cancel{\frac{V_4^2}{2g}} + \cancel{z_4} - H_L + H_p$$

$$H_L = H_p$$

by definition (only due to the valve) $H_L = \frac{kv^2}{2g} = \frac{kv_3^2}{2g}$

$$H_L = H_p = \frac{k V_3^2}{2g} = 100 - 100 Q_1$$

$$Q = 0.2 + Q_3$$

$$= \frac{k}{2g}\left(\frac{Q_3}{A}\right)^2 = \frac{k}{2g}\left(\frac{Q_3}{\frac{\pi}{4}D_3^2}\right)^2 = 100 - 100 Q_1$$

$$\frac{k}{2g}\left(\frac{Q_3}{\frac{\pi}{4}D_3^2}\right)^2 = 100 - 100(0.2 + Q_3)$$

$$165 Q_3^2 = 100 - 20 - 100 Q_3$$

$$165 Q_3^2 = 80 - 100 Q_3$$

$$165 Q_3^2 + 100 Q_3 - 80 = 0$$

$$Q_3 = 0.456 \text{ m}^3/\text{s}$$

$$Q_1 = 0.2 + Q_3 = 0.2 + 0.456 = 0.656 \text{ m}^3/\text{s}$$

Other Answers:

2. $F_V = 2279.5$ lbf $\quad x_1 = 3.30$ ft
$F_H = 7987.2$ lbf $\quad y_1 = 3.77$ ft

3. by trial and error $D_2 \approx 1.72$ m

4. $v_2 = \sqrt{2gh}$, $a = \frac{1}{m}(-F + 2gh\rho A)$

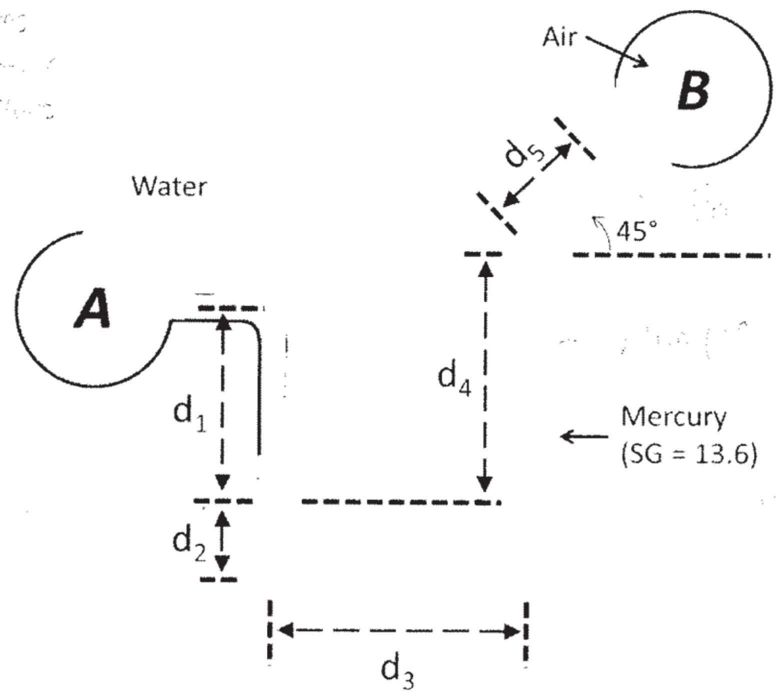
6. $u_\theta(r) = \left(\omega_1 - \frac{R_0^2(\omega_0 - \omega_1)}{R_1 - R_0^2}\right) r + \left(\frac{R_0^2 R_1^2(\omega_0 - \omega_1)}{R_1^2 - R_0^2}\right)\frac{1}{r}$

where $u_r = 0 \quad u_z = 0$

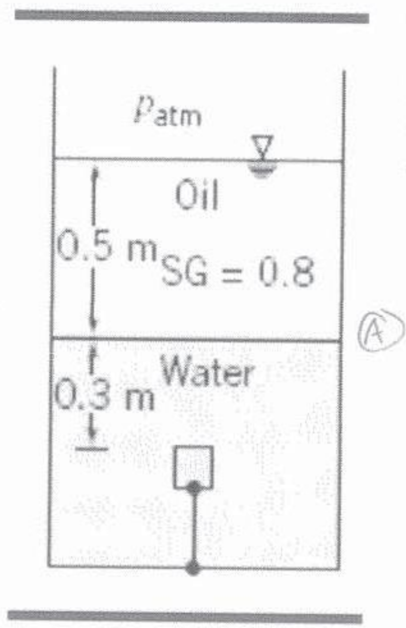
15/15

NICE.
cm

1. Find the difference in pressure between tanks *A* and *B*, if d_1 = 2 ft, d_2 = 6 in, d_3 = 2 ft, d_4 = 2.4 in, and d_5 = 4 in. [5 points] (Figure not to scale)



2. A 512-mL cube of solid oak (SG = 0.77) is held submerged by a tether as shown. Calculate (a) the actual force of the water on the bottom surface of the cube and (b) the tension in the tether. [10 points]



Assumptions
(1) static
(2) incompressible fluids
(3) incompressible block
(4) ignore the weight of the string

(b) $\sum F_z = 0 = F_B + T - w$

$= 5.02N - T - mg$

$= 5.02N - T - \rho_w SG_{oak} V_{oak} g$

$T = 5.02N - \rho_w SG_{oak} V_{oak} g$

$= 5.02N - (999 \frac{kg}{m^3})(0.77)(0.08m)^3(9.81 \ldots$

$= 5.02N - 3.86N$ 5/5

$\boxed{T = 1.16 N}$ (b)

Actual force of water on bottom of block is $F_{P_2} = (P_A + \rho g h_2)A$ $h_2 = 0.38m$

$P_A = (P_{atm} + SG_{oil}\, \rho_w\, g\, h_{oil})$

$\{(101300 Pa) + (0.8)(999 \frac{kg}{m^3})(9.81 m/s^2)(0.5m)\}$

$= 105.2$ kPa

P at $h_2 = 105.2 kPa + \rho g_w h_2$

$= 105.2 kPa + (999 \frac{kg}{m^3})(9.81 m/s^2)(\ldots$

P at $h_2 = 108.9$ kPa

$F_{acting\ on\ bottom\ of\ block} = (108.9 kPa)(0.08)^2$

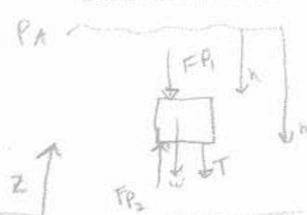
$= 0.697$ kN

5/5 $\boxed{= 697 N}$ (a)

1 mL = 1 cm³

512 mL = 512 cm³ = V_{block}

$Side_{block} = \sqrt[3]{512 cm^3} = 8cm = 0.08m$



$F_{P_1} = (P_A + \rho g h_1) A$

$F_{P_2} = (P_A + \rho g h_2) A$

$F_{P_2} - F_{P_1} = P_A A + \rho g h_2 A - P_A A + \rho g h_1 A$

$= \rho g (h_2 - h_1) A = \rho g V = F_B$

At the surface of the water $\underline{P_A = P_{atm} + SG_{oil}\, \rho_w g\, h_{oil}}$

$\sum F_{net,z} = F_B + T - w = 0$

F_B = Buoyancy of block in water

$F_B = \rho g V$

$F_B = (999 \frac{kg}{m^3})(9.81 m/s^2)(0.08 m)^3$

$\boxed{F_B = 5.02 N}$ upwards

Department of Chemical Engineering
McGill University
CHEE 314 Fluid Dynamics

Midterm | Fall 2014

STUDENT NAME: ____________

STUDENT NUMBER: ____________

Regular Instructions:

Time: 1 hour, 50 minutes (9:35 AM to 11:25 AM)

There are **FIVE** questions; each question is worth 10 points. (Total = 50 points)

Allowable aids: Calculator, dictionary, 10 pages of hand-written notes.

If you finish before 11:15 AM, you may leave.
If you finish after 11:15 AM, please remain seated out of consideration for your classmates.

REALLY IMPORTANT instructions:

Quickly scan through all the problems before starting.

Then: Read the problem carefully. Think about what it is asking.

Then think about it again.

If you don't know where to start, go back to the problem solving framework:

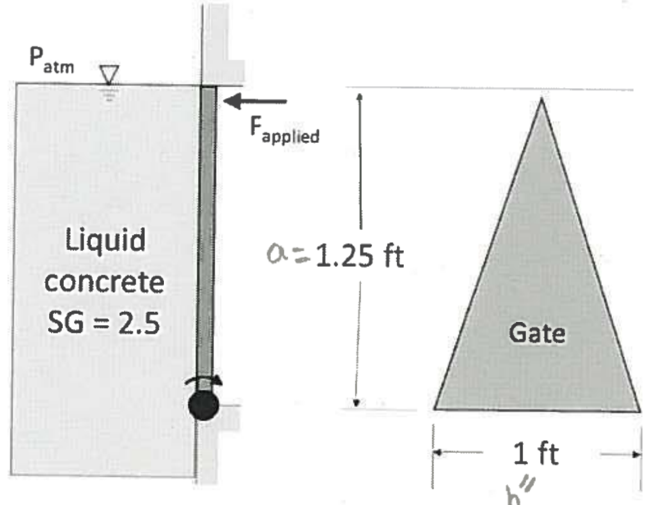
1) draw & state assumptions; 2) define physics; 3) where? ; 4) solve;
If appropriate, 5) check units + assumptions

Finally:

Relax. Don't panic. Breathe. You've got this.

~~7/10~~ 8

1. A triangular access gate must be provided in the side of a container containing liquid concrete, as shown. Calculate the force $F_{applied}$ at the top of the gate, required to keep the gate closed.



- Incompressible

$\frac{w}{y} = \frac{b}{a}$

$w = y\, b/a$

Balance moments

- From liquid

$y' F_R = \int y p\, dA$ $\qquad A = \frac{yw}{2} = \frac{y^2 b}{2a}$

$y' F_R = \int y (\rho g y)(\frac{yb}{a})\, dy$ $\qquad dA = \frac{yb}{a} dy$ $\qquad p = \rho g y$

$= \frac{\rho g b}{a} \int_0^a y^3 dy$

- Patm on both the sides so it cancels

$= \frac{\rho g b}{4a}(a^4) = \frac{\rho g b a^3}{4}$

p should be $\rho g (a-y)$ messed up

$\frac{\rho g b a^3}{4} = F_{applied} \cdot a$ $\quad$ h-y'

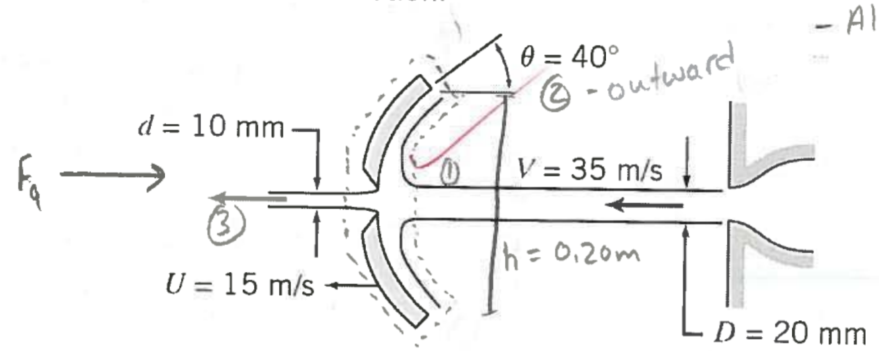
$F_{app} = \frac{\rho g b a^2}{4}$ $\qquad [\frac{lbm}{ft^3}][\frac{ft}{s^2}][ft][ft^2] \Rightarrow \frac{lbm \cdot ft}{s^2} = lbf$

$\boxed{F_{app} = 61\ lbf}$

10

2. The circular dish, whose cross section is shown, has an outside diameter of 0.20 m. A water jet with speed of 35 m/s strikes the dish concentrically. The dish moves to the left at 15 m/s. The jet diameter is 20 mm. The dish has a hole at its center that allows a stream of water 10 mm in diameter to pass through without resistance. The remainder of the jet is deflected and flows along the dish. Calculate the force required to maintain the dish motion.

- Already Developed



water $\rho = 1000\ Kg/m^3$

$\leftarrow +$

Assumptions
1) incompressible
2) Fluid speed remains constant
3) Fluid mass in Disk neglected
4) ignore gravity

- Dish moves at $U = 15 m/s$ cst ∴ $F_{net} = 0$

$V' = V - u$
$[V' = 20\ m/s]$

$$-F_a = \cancel{F_{Sx}} + \cancel{F_{Bx}} = \cancel{\frac{\partial}{\partial t}\int_{CV} \rho \vec{V} d\forall}^{\,0} + \int_{CS} \vec{V} \cdot \rho \vec{V} dA$$

$$-F_a = -\int V_1' \rho V_1' dA_1 + \int -V_2' \rho V_2' dA_2 \cos\theta + \int V_3' \rho V_3' dA_3$$

$$V_1' = V_2' = V_3' = V' = 20 m/s$$

Get A_2 - where A_2 is the entire area around the disk

$\frac{dM}{dt} = 0$

$$V'A_1 = V'A_2 + V'A_3$$
$$A_2 = A_1 - A_3$$
$$= 2.3562 \times 10^{-4}\ m^2$$

$A_1 = \frac{\pi D^2}{4} = 3.1416 \times 10^{-4}$

$A_4 = \frac{\pi d^2}{4} = 7.854 \times 10^{-5}$

$$-F_a = -V'^2 A_1 \rho - V'^2 A_2 \cos\theta \rho + V'^2 A_3 \rho = -V'^2 A[1 + \cos\theta]\rho$$

$-F_a = -166.4 N$

$[F_a = 166.4 N]$ to the right

10

3. A smuggler is carrying an illegal shipment of gold in his boat. In order to cross a dam in the river, the boat floats in a large bucket, which is then picked up by heavy machinery and carried across the dam. During the crossing, the smuggler realizes the police have spotted him. Quickly, he dumps the gold out of his boat, into the bucket. Does the level of water **in the bucket** go up or down? State any assumptions, show your work.

- assuming no loss of water
- M_B includes mass of evil Criminal

M_G = mass gold $= \rho_G V_G$

M_B = mass Boat $= \rho_B V_B$

+1

without Boat + Gold

+2

Balance of Forces initially

For Boat

$M_B g = \rho g V_1$

$V_1 = \frac{M_B}{\rho_w}$

Balance of Forces after

$(M_B) g = \rho_w g V_2$

$V_2 = \frac{M_B}{\rho_w}$

DV_2 = Level Change $= V_2 + V_G$

For Gold in Boat

+2

$M_G g = \rho_w g V_{1G}$

$V_{1G} = \frac{M_G}{\rho_w}$

$DV_1 = V_1 + V_{1G}$

- Assuming $\rho_{Gold} > \rho_w$ (which it is obviously)

for Water level to stay same

+2

$DV_1 = DV_2$ $\qquad V_1 = \frac{M_B}{\rho_w} = V_2$

$\cancel{V_1} + V_{1G} = \cancel{V_2} + V_G$

$V_{1G} = V_G$ $\rightarrow$ not the case as $V_{1G} > V_G$

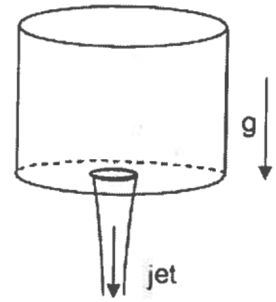
- since $\rho_G > \rho_w$, the weight of gold in water is a larger volume so $DV_1 > DV_2$

$\therefore$ The water level will lower (Down)

+3

10

4. Water drains out of the bottom of a barrel through a filter, and exits to atmosphere, forming a jet of circular cross-section. The jet tapers from a diameter of 20 mm to a diameter of 10 mm over a vertical distance of 50 cm.

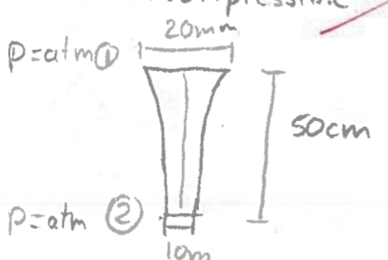


[Note: the jet of water is exposed to P_{atm} on all sides]

a) Find the flow rate, stating all relevant assumptions.

b) The barrel has a diameter of 1 m. If the barrel is filled with soda (mixture of CO_2 gas and water; SG = 0.7), and the filter removes all the CO_2 gas, what is the rate of change of liquid level in the barrel? [Hint: assume density inside barrel = constant].

1/2 a) - Jet follows a streamline — frictionless
- Incompressible



$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gh_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gh_2^{\,0}$$ 2

$A_1 = 3.1416\times10^{-4}\,m^2$

$A_2 = 7.854\times10^{-5}\,m^2$

$h_1 = 50cm = 0.5m$

$$\frac{Q^2}{2A_1^2} + gh_1 = \frac{Q^2}{2A_2^2}$$

1 $Q_1 = V_1A_1 \quad V_1 = Q/A_1$ 1

$Q_2 = V_2A_2 \quad V_2 = Q/A_2$

$Q_1 = Q_2$

$$Q^2\left[\frac{1}{2A_2^2} - \frac{1}{2A_1^2}\right] = gh_1$$

$$Q = \sqrt{\frac{gh_1}{\left[\frac{1}{2A_2^2} - \frac{1}{2A_1^2}\right]}}$$

$$\left[Q = 2.54\times10^{-4}\,m^3/s\right]$$ 2

b) - assume cst density inside barrel

if $Q = 2.54\times10^{-4}\,m^3/s$ → rate of water leaving SG = 1

- inside Barrel = 0.7

→ assuming filter removal of CO_2 does not interfere with H_2O flow out

- Water accounts for 70% of Vol inside
- Mass CO_2 negligible

$SG_i\,Q_{in} = SG_{out}\,Q_{out}$

$Q_{in} = \frac{SG_{out}\,Q_{out}}{SG_{in}}$ 1

$Q_{in} = 3.63\times10^{-4}\,m^3/s$

$\frac{d\forall}{dt} = \frac{dh}{dt}\frac{\pi D^2}{4} = Q_{in}$ 1

$\frac{dh}{dt} = \frac{4Q_{in}}{\pi D^2}$

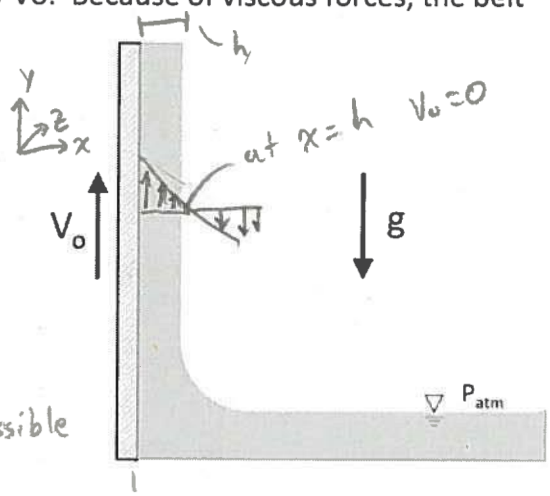
1 $\left[\frac{dh}{dt} = 4.62\times10^{-4}\,m/s = 0.462\,mm/s\right]$

5. A wide moving belt passes through a container of a viscous liquid. The belt moves vertically upward with a constant velocity Vo. Because of viscous forces, the belt picks up a film of fluid of thickness *h*. Gravity tends to make the fluid drain down the belt. Assume that the flow is laminar, steady, and fully-developed.

Derive an expression for the velocity distribution in the fluid film, as it is dragged up the belt.

~~4.5/10~~

4/10



1) laminar

2) 2D $\partial/\partial z = 0$

3) Steady $\partial/\partial t = 0$

4) fully-Developed $\frac{\partial \vec{V}}{\partial y} = 0$

5) Incompressible

$\underline{V} = (0, v, 0)$

Continuity $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ $\therefore \frac{\partial u}{\partial x} = 0$ $u(x) = cst$

at $x=0$ $u=0$

$\therefore u(x) = 0$ ✓ 2

Navier-Stokes $\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\nabla^2 v - \rho g$

$0 = \cancel{\frac{\partial p}{\partial y}} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) - \rho g$

0

$\int \frac{\partial p}{\partial y} + \rho g \, dx = \int \mu \frac{\partial^2 v}{\partial x^2} dx$ $\frac{\partial p}{\partial y} = 0$

$\int C_1 + \left(\frac{\partial p}{\partial y} + \rho g\right)x \, dx = \int \mu \frac{\partial v}{\partial x} dx$

$\mu V(x) = \frac{1}{2}\left(\frac{\partial p}{\partial y} + \rho g\right)x^2 + C_1 x + C_2$

1.0 $V(x) = \frac{\left(\frac{\partial p}{\partial y} + \rho g\right)}{2\mu}x^2 + C_1 x + C_2$

B.Cs ✓ 1

at $x = 0$ $V = V_0$

$V_0 = C_2$ ✓

at $x = h$ $V = 0$ ✗

$C_1 h = -\left(\frac{\partial p}{\partial y} + \rho g\right)\frac{h^2}{2\mu} - V_0$

$C_1 = -\frac{\left(\frac{\partial p}{\partial y} + \rho g\right)h}{2\mu} - \frac{V_0}{h}$

$$\left[V(x) = \frac{\left(\frac{\partial P}{\partial y} + \rho g\right)}{2\mu}x^2 - \frac{\left(\frac{\partial p}{\partial y} + \rho g\right)h}{2\mu}x - \frac{V_0}{h}x + V_0\right]$$

H = max height

where $p = \rho g z = \rho g(H-y) + P_0$

$\frac{\partial p}{\partial y} = -\rho g$

$V(x) = -\frac{V_0}{h}x + V_0$

$$\left[V(x) = V_0\left(1 - \frac{x}{h}\right)\right]$$ 1

10

CHEE 314: Fluid Mechanics **Quiz #2 (2014)**

1. A 'super-soaker' water gun is pumped to create a static pressure in the water tank. When the trigger is pressed, water flows steadily through a 3 cm diameter pipe, and discharges through a 0.3 cm diameter nozzle, to atmospheric pressure. The flow rate is 10 liters per minute. Calculate the minimum static gage pressure required in the pipe to produce this flow rate. State any assumptions made.

[5 points]

Assume
1) steady
2) incompressible
3) inviscid
4) along a streamline

3cm = 0.03m
0.3cm = 0.003 m
$1\,L = 1000\,cm^3 \left(\frac{1\,m}{100}\right)^3 = 0.001\,m^3$

Conservation of mass: $0 = \frac{d}{dt}\int \rho\, dA + \int v\rho\, dA$

$$0 = -V_1 A_1 + V_2 A_2$$
$$V_1 A_1 = V_2 A_2 = Q$$

$V_1 = \frac{Q}{A_1}$

$$\underbrace{10\frac{L}{min}\left(\frac{1\,min}{60\,s}\right)}_{Q}\underbrace{\left(\frac{4}{\pi}\right)\left(\frac{1}{0.03}\right)^2_{m}\left(\frac{0.001\,m^3}{1\,L}\right)}_{1/A_1} = \boxed{0.236\ m/s = V_1}$$

$$V_2 = \frac{Q}{A_2} = \underbrace{\left(10\frac{L}{min}\right)\left(\frac{1\,min}{60\,s}\right)}_{Q}\underbrace{\left(\frac{1}{0.003}\right)^2_{m}\left(\frac{0.001\,m^3}{L}\right)\left(\frac{4}{\pi}\right)}_{1/A_2}$$

Use bernolli along the center streamline

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + g\cancel{z_1} = \frac{\cancel{P_2}}{\rho} + \frac{V_2^2}{2} + \cancel{gz_2}$$

$z_1 = z_2$
$P_1 = ?$
$P_2 = P_{atm}$

$$\frac{V_1^2}{2} + \frac{P_1}{\rho} = \frac{V_2^2}{2}$$

$$P_1 = \left(\frac{V_2^2 - V_1^2}{2}\right)\rho$$

$$P_1 = \left(\frac{23.6\,m/s^2 - (0.236\,m/s)^2}{2}\right)1000\,\frac{kg}{m^3}$$

$P_1 = 277940\ Pa$ = $\boxed{277.9\ kPa\ \text{(gauge) required to produce the flowrate of 10 L}}$

5

2. A thin plate of area A is located so that it is normal to a moving stream of fluid. Assume that the drag, d (with dimensions MLT^{-2}) that the fluid exerts on the plate is a function of A, the fluid viscosity μ, fluid density ρ, and fluid velocity V of the fluid approaching the plate.

[5 points]

a) How many Pi groups can be found?

b) Determine the PI groups that relate drag d with fluid velocity V

c) **<u>IF</u>** the relationship was found to be: $\frac{\rho}{\mu} d = \text{fcn}\left(\frac{\sqrt{A}\,\rho\,V}{\mu^2}\right)$, and our model system kept the ρ and V parameters constant, but increased μ by a factor of 10; how would the area have to change to obtain similar drag forces between model and prototype? [This relationship is NOT the correct Pi groups found in (b)]

$d = \text{fcn}(A, \mu, \rho, V)$

$d = MLT^{-2}$
$A = L^2$
$\mu = ML^{-1}T^{-1}$
$\rho = ML^{-3}$ $n = 5$
$V = LT^{-1}$

$$\begin{array}{c|c} & \begin{array}{ccccc} d & A & \mu & \rho & V \end{array} \\ \begin{array}{c} M \\ L \\ T \end{array} & \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 1 & 2 & -1 & -3 & 1 \\ -2 & 0 & -1 & 0 & -1 \end{bmatrix} \end{array}$$

rank = 3 = m

(a) # of pi groups = # parameters − rank of matrix
$i = n - m$
$i = 5 - 3$
$i = 2$ pi groups can be made

1

b) choose A, μ, ρ as repeating parameter:

$\Pi_1 = A^a \mu^b \rho^c d$
$= (L^2)^a (ML^{-1}T^{-1})^b (ML^{-3})^c (MLT^{-2}) = M^0L^0T^0$

M: $b + c + 1 = 0$
L: $2a - b - 3c + 1 = 0$
T: $-b - 2 = 0 \Rightarrow b = -2$
$-2 + c + 1 = 0 \Rightarrow c = 1$
$2a + 2 - 3 + 1 = 0$
$2a = 0 \Rightarrow a = 0$

$$\boxed{\Pi_1 = \frac{\rho d}{\mu^2}}$$

$\Pi_2 = A^a \mu^b \rho^c V$
$(L^2)^a (ML^{-1}T^{-1})^b (ML^{-3})^c (LT^{-1}) = M^0L^0T^0$

M: $b + c = 0$
L: $2a - b - 3c + 1 = 0$
T: $-b - 1 = 0 \Rightarrow b = -1$
$-1 + c = 0 \Rightarrow c = 1$
$2a + 1 - 3 + 1 = 0$
$2a = +1$
$a = 1/2$

$$\boxed{\Pi_2 = \frac{\sqrt{A}\,\rho\,V}{\mu}}$$

3

continued on back

(c) $\frac{\rho}{\mu} d = fcn\left(\frac{\sqrt{A}\rho V}{\mu^2}\right)$

$\mu_{model} = 10\mu_{prototype}$

$\rho_m = \rho_p$

$V_m = V_p$

#1 B

To obtain similar drag forces, since d is a function of $\frac{\sqrt{A}\rho V}{\mu^2}$, this term must remain constant.

$$\Pi_{model} = \Pi_{prototype}$$

$$\frac{\sqrt{A_m}\,\rho_m V_M}{(\mu_m)^2} = \frac{\sqrt{A_p}\,\rho_p V_p}{(\mu_p)^2}$$

$$\frac{\sqrt{A_m}\,\cancel{\rho_p V_p}}{(10\cancel{\mu_p})^2} = \frac{\sqrt{A_p}\,\cancel{\rho_p V_p}}{\cancel{(\mu_p)^2}}$$

$$\frac{\sqrt{A_m}}{(10)^2} = \sqrt{A_p}$$

$$\sqrt{A_m} = 100\sqrt{A_p}$$

$$\boxed{A_m = 10000 A_p}$$

So to keep the drag similar, while increasing μ 10-fold and keeping ρ, V constant, the area of the model must be 10000-fold bigger than the area of the prototype.

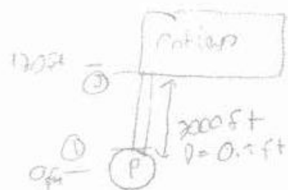
9.3

1. Your friend asks you to pick a pump, to deliver water from a lake to their cottage. The cottage is 120 ft above the surface of the lake. The water will need to travel through 2000 feet of 6-inch diameter cast iron pipe. A flow rate of 3 ft^3/s is needed at the exit. What head does the pump have to contribute? You may neglect minor losses (DO NOT neglect major losses).

Info: roughness (ε) of cast iron = 0.00085; ν of water = 1.076×10^{-5} ft^2/s

[5 points]

5



$Q = 3 ft^3/s = \bar{V} A \Rightarrow \bar{V} = \frac{Q}{A} = \frac{3 ft^3/s}{\frac{\pi (0.5)^2}{4}} = \boxed{15.27 ft/s}$

$Re = \frac{\bar{V} D}{\nu} = \frac{(15.27 ft/s)(0.5 ft)}{1.076 \times 10^{-5} ft^2/s} = 709984$

$\frac{\varepsilon}{D} = \frac{0.00085}{0.5} = 0.0017$

f found on moody chart ≈ 0.023

$\left(\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1\right) - \left(\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2\right) = h_{major} + \cancel{h_{minor}}^{0} - h_{pump}$

Assume
$P_1 = P_2$ ok, but $V_1 \approx 0$
$V_1 = V_2$ since A_1 A_2 and Q = constant
$V_2 = v_{pipe}$

8

$h_{pump} = h_{major} + (z_2 - z_1)$

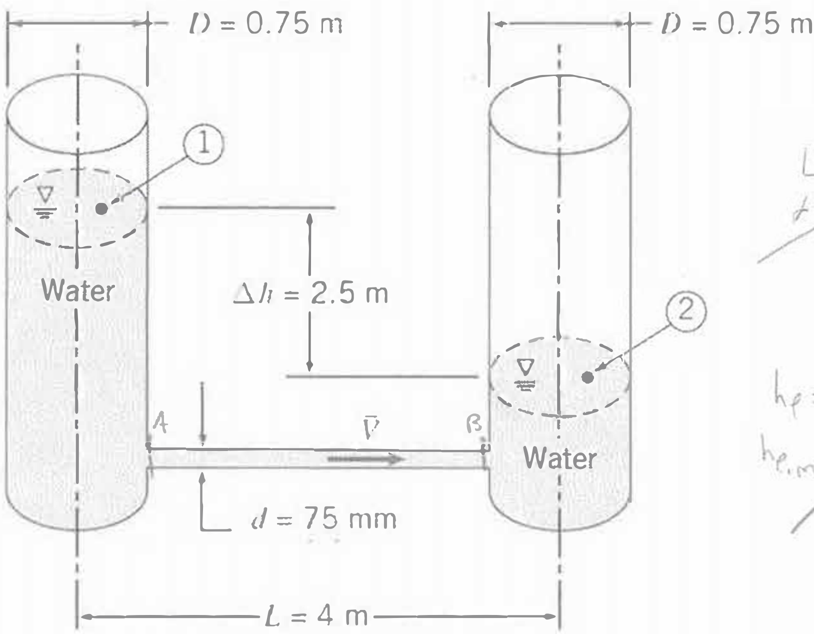
$h_{pump} = f \frac{\bar{V}^2}{2} \frac{L}{D} + 120 ft - 0 ft$

$h_{pump} = (0.023)\left(\frac{(15.27 ft/s)^2}{2(32.174 ft/s^2)}\right)\left(\frac{2000 ft}{0.5 ft}\right)$

$\boxed{h_{pump} = 439.8 ft}$

2. Two connected cylinders, open to atmosphere at the tops, are connected by a straight tube as shown. For the instant shown, estimate the rate of change of water level in the left cylinder. Assume $K_{entrance} = 0.5$, $K_{exit} = 1$, and all surfaces have negligible roughness.

4.5 **[5 points]**



~~$L = 4m - 0.75m = 3.25m$~~

~~$d = 0.075$ m~~

~~$\frac{\varepsilon}{D} = 0 \rightarrow$ smooth~~

~~$h_f = f\left(\frac{L}{D}\right)\left(\frac{\bar{V}^2}{2}\right) = 43.33 f\left(\frac{\bar{V}^2}{2}\right)$~~

~~$h_{e,m} = (0.5+1)\left(\frac{\bar{V}^2}{2}\right) = 1.5\frac{\bar{V}^2}{2}$~~

~~$\bar{V} = \frac{Q}{A}$~~

~~$A = \frac{\pi(0.075)^2}{4} = 0.00442$~~

Find Q, and we can find $\frac{dh}{dt}$

~~$\left(\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1\right) - \left(\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2\right) = h_f + h_{em}$~~

~~assume $P_1 = P_2$~~

~~Since each pipe cylinder is the same diameter, $V_1 = V_2$~~

~~$2.5m = \frac{1.5}{2}\left(\frac{Q}{A}\right)^2 + \frac{43.33}{2} f \left(\frac{Q}{A}\right)^2$~~

~~$2.5m = 38390\, Q^2 + 1109041 (f)(Q^2)$~~

~~$Q = \frac{2.5}{38390 + 1109041 f}$~~

~~Guess 1 $f = 0.03 \Rightarrow Q = 0.0000348 \frac{m^3}{s}$~~

~~$V = \frac{Q}{A} = 0.00789$ m/s~~

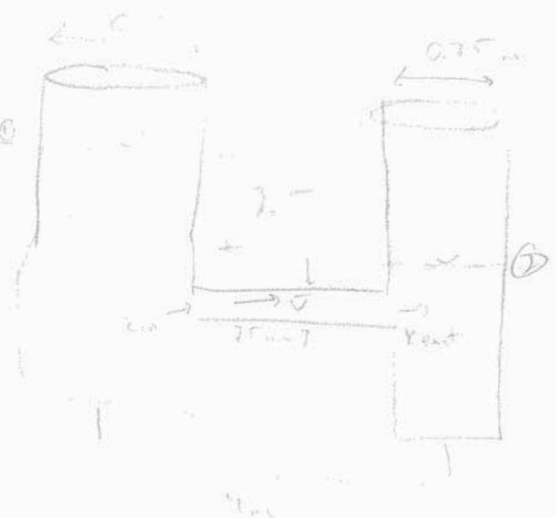
~~$Re = \frac{VD}{\nu} = 591.9 \Rightarrow f = \frac{64}{Re}$~~

~~$f = 0.101$~~

~~$Re = 28.3 \Leftarrow V = \frac{Q}{A} = 0.00375$ m/s $\Leftarrow Q = 0.0000165 \frac{m^3}{s}$~~

~~$f = 0.2275 \Rightarrow Q = 0.000086 \Rightarrow V = 0.0019 \Rightarrow Re = 145.9 \Rightarrow f = 0$~~

$\boxed{\text{See Back}}$ →



$\frac{\varepsilon}{D} = 0$

$L = 3.25\,m$

$A = \frac{\pi(0.075)^2}{4} = 0.00442\,m^2$

$k_{in} = 0.5$

$k_{ext} = 1$

$P_1 = P_2$

$V_1 = V_2$ since same diameter so cylinder height change ...

$\left(\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + z_1\right) - \left(\cancel{\frac{P_2}{\rho g}} + \cancel{\frac{V_2^2}{2g}} + z_2\right) = h_{major} + h_{minor}$

$z_1 - z_2 = 2.5\,m = f\frac{L}{D}\frac{V^2}{2g} + 0.5\frac{V^2}{2g} + 1.0\frac{V^2}{2g} + 0.5 \quad V = \frac{Q}{A}$

$2.5\,m = f(43.333)V^2 + 0.75\,V^2$

$\left[\frac{2.5\,m\,2g}{f(43.333) + 0.75}\right]^{1/2} = V$ ... guess ... $f = 0.01$ $\quad Re = \frac{VD}{\nu}$

	f	V (m/s)	Re
#1	0.01	1.45	150000
#2	0.025	1.39	1043479
#3	0.0116	1.41	

Found f ... Moody chart with $\frac{\varepsilon}{D} = 0$ and ... the Re found in the previous step

$\left|\frac{V_3 - V_2}{V_3}\right| = 0.015 \rightarrow$ ...

(*) I may have used the wrong D in the Re when ... (*)

$V = 1.41\,m/s = \frac{Q}{A_{pipe}} \Rightarrow Q = V A_{pipe} = (1.41\,m/s)(0.00442\,m^2) = 0.00623\,\frac{m^3}{s} = \dot{V}$

$\frac{dh}{dt} = \frac{Q}{A} = \frac{0.00623}{\frac{\pi(0.75)^2}{4}} = \boxed{-0.0141\,\frac{m}{s}}$

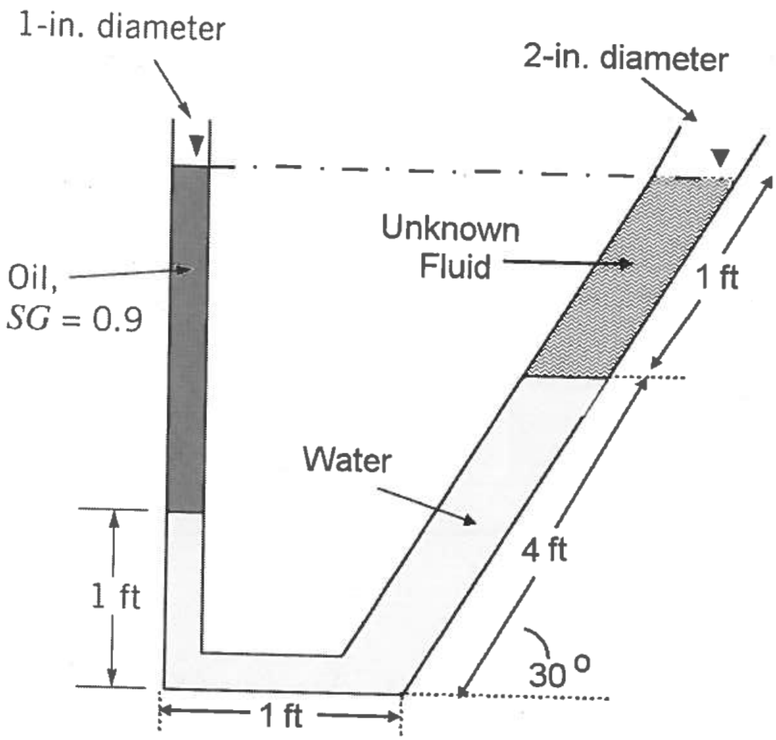
$\rightarrow$ the height is decreasing, since Q is leaving, at a rate of $0.0141\,\frac{m}{s}$

CHEE 314: Fluid Mechanics **Quiz #1 (2015)**

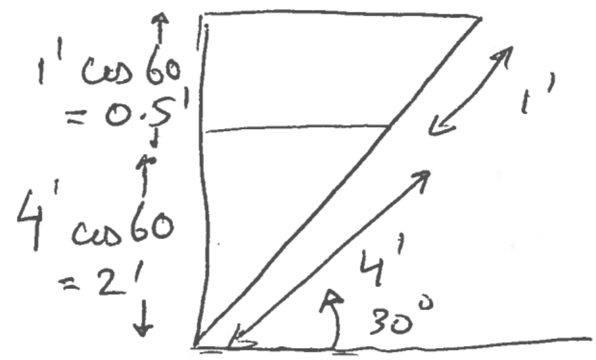
STUDENT NAME: Solutions **STUDENT #:** —

Q1. Water, oil, and an unknown fluid are contained in the tubes shown. Determine the density of the unknown fluid. [Drawing not to scale]

[5 points]



- ONLY VERTICAL HEIGHT MATTERS!



- Balance pressure on left vs. right.

$$\rho_{H_2O}\, g(1') + \rho_{OIL}\, g(1.5') = \rho_{H_2O}\, g(2') + \rho_{?}\, g(0.5')$$

⋮ Recall $\rho_X = SG_X \, \rho_{H_2O}$.

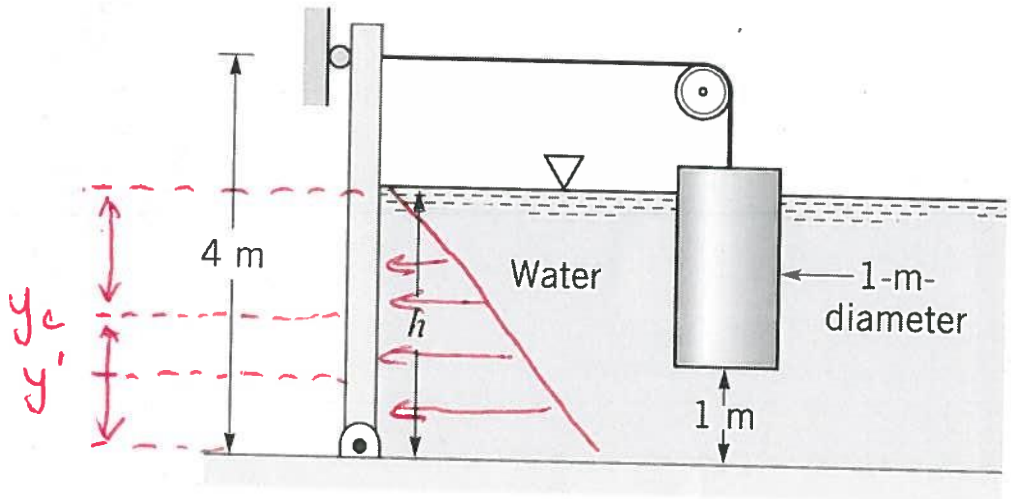
Solve: $SG_? = 0.7$

$\therefore \rho_? = 0.7 \times \rho_w =$ $\boxed{\begin{array}{l} 700\ kg/m^3 \\ 1.358\ slugs/ft^3 \\ 43.6\ lbm/ft^3 \end{array}}$

CHEE 314: Fluid Mechanics **Quiz #1 (2015)**

Q2. A 1-m diameter cylindrical mass, M, is connected to a 2-m wide rectangular gate as shown in the figure. The gate is to open (clockwise) when the water level, h, drops below 2.5 m. Determine the required value for M. Neglect friction at the gate hinge and the pulley.

[5 points]



APPROACH:

- CALCULATE M @ HINGE DUE TO WATER.
- CALCULATE M @ HINGE DUE TO MASS.
- DON'T FORGET BUOYANCY of MASS

$$F_R = \rho g h_c A = \rho g h^2 w$$

$$y' = \left[\text{USE YOUR FAVOURITE METHOD}\right] = \frac{1}{3} h \left(\text{FROM BOTTOM! (HINGE)}\right)$$

$$\therefore M = \frac{1}{3} \rho g h^3 w \text{ @ HINGE DUE TO WATER.}$$

$$\text{Tension} = Mg - F_{Buoyancy} = Mg - \rho_w g V_{cylinder}$$

$$\text{Balance: } \frac{1}{3} \rho_w g h^3 w = 4\left(Mg - \rho_w g V_{sub}\right)$$

CALCULATE

$$M = 2480 \text{ kg.}$$

CHEE 314: Fluid Mechanics **Quiz #2 (2015)**

STUDENT NAME: ______________________ **STUDENT #:** ______________

Q1. The *x*-component of velocity in a steady, incompressible flow field in the *x*-*y* plane is $u = A/x$, where $A = 2\ m^2/s$, and *x* is measured in meters. Find the simplest *y* component of velocity for this flow field.

[3 points]

$$u = \frac{A}{x}$$

the continuity eqⁿ for an incompressible, steady flow field is — given

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \cancel{=0} + \underbrace{\cancel{\frac{\partial w}{\partial z}}}_{0 \text{ (no flow in z-direction) } \cancel{\text{assump}}} = 0$$

3.0

7.0

10

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x}$$

$$= -\left(-\frac{A}{x^2}\right)$$

$$= \frac{A}{x^2}$$

∴ Integrating, we get,

$$v = \frac{A}{x^2} \cdot y + C_1$$

for the simplest y component, we can assume C_1 to be zero.

∴ simplest v is, $\boxed{v = A \cdot \frac{y}{x^2}}$

A is given as $2 m^2/s$

$$\boxed{\frac{\cancel{m^2}}{s} \times \frac{m}{\cancel{m^2}} = \frac{m}{s}}$$

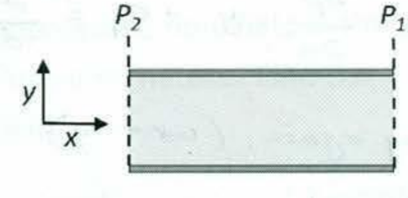
$$\therefore \boxed{v = 2 \cdot \frac{y}{x^2}}$$

(units - m/s)

x, y in meters multiplied with 2 (m^2/s)

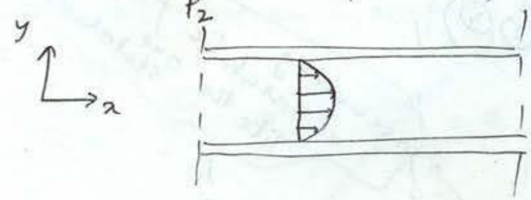
CHEE 314: Fluid Mechanics **Quiz #2 (2015)**

Q2. A liquid is contained between two flat, fixed plates. A pressure gradient applied along the x-axis drives laminar flow (i.e., P2 > P1). Assume that the flow is fully-developed and steady. Derive an expression for fluid velocity between the plates. State any assumptions.



[Tip: sketch an expected flow profile] [7 points]

the plates are flat and fixed. so, no ~~p~~-slip condition will apply in the boundaries. so an expected flow profile may look like —



Assumptions:
1) let's assume the fluid is incompressible (in order to be able to apply N-S)
2) 2-D flow. ($w=0$ & $\frac{\partial}{\partial z}=0$)
3) g only works in negative y-direction
4) steady flow.
5) fully developed flow ($\frac{\partial}{\partial x}=0$)
6) from simplified continuity ($\frac{\partial v}{\partial y}=0$)
7) $v=0$ (from ~~(4)~~ eq^n(5))

Lets write the continuity equation and the Navier/stokes equations for this field -

for an incompressible fluid,

$$\cancelto{(5)}{\frac{\partial u}{\partial x}} + \frac{\partial v}{\partial y} + \cancelto{(2)}{\frac{\partial w}{\partial z}} = 0 \quad \text{-eq}^n(1)$$

N-S

$$\rho\left(\cancelto{4}{\frac{\partial u}{\partial t}} + \cancelto{5}{u\cdot\frac{\partial u}{\partial x}} + \cancelto{7}{v\cdot\frac{\partial u}{\partial y}} + \cancelto{(2)}{w\cdot\frac{\partial u}{\partial z}}\right) = \cancelto{(3)}{\rho g_x} - \frac{\partial p}{\partial x} + \mu\left(\cancelto{5}{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \cancelto{(2)}{\frac{\partial^2 u}{\partial z^2}}\right) \quad \text{eq}^n(2)$$

$$\rho\left(\cancelto{4}{\frac{\partial v}{\partial t}} + \cancelto{5}{u\cdot\frac{\partial v}{\partial x}} + \cancelto{(6)}{v\cdot\frac{\partial v}{\partial y}} + \cancelto{(2)}{w\cdot\frac{\partial v}{\partial z}}\right) = \rho g_y - \frac{\partial p}{\partial y} + \mu\left(\cancelto{4}{\frac{\partial^2 v}{\partial x^2}} + \cancelto{(6)}{\frac{\partial^2 v}{\partial y^2}} + \cancelto{(2)}{\frac{\partial^2 v}{\partial z^2}}\right) \quad \text{eq}^n(3)$$

$$\rho\left(\cancelto{(2)}{\frac{\partial w}{\partial t}} + \cancelto{(2)}{u\cdot\frac{\partial w}{\partial x}} + \cancelto{(2)}{v\cdot\frac{\partial w}{\partial y}} + \cancelto{(2)}{w\cdot\frac{\partial w}{\partial z}}\right) = \cancelto{(3)}{\rho g_z} - \frac{\partial p}{\partial z} + \mu\left(\cancelto{(2)}{\frac{\partial^2 w}{\partial x^2}} + \cancelto{(2)}{\frac{\partial^2 w}{\partial y^2}} + \cancelto{(2)}{\frac{\partial^2 w}{\partial z^2}}\right) \quad \text{eq}^n(4)$$

eq^n (1) reduces to, $\frac{\partial v}{\partial y} = 0$ ...

$\therefore$ $v=$ constant (by integration)

at the boundaries, $v=0$ $\therefore$ constant $=0$

$\therefore v=0$ everywhere ... (5)

eq^n (3) reduces to $\frac{\partial p}{\partial y} = \rho g_y$

Lets neglect gravity

eq^n (2) reduces to, $\frac{\partial p}{\partial x} = \mu\cdot\frac{\partial^2 u}{\partial y^2}$ as, $\frac{\partial}{\partial x} = \frac{\partial}{\partial z} = 0$ (because, fully developed and 2-D)

so, $\frac{\partial^2 u}{\partial y^2} = \frac{d^2 u}{dy^2}$

P. T. O.

$$\text{so, } \frac{\partial p}{\partial x} = \mu \cdot \frac{d^2 u}{dy^2} \Rightarrow \frac{1}{\mu}\cdot\frac{\partial p}{\partial x} = \frac{d^2u}{dy^2}$$

integrating once, (w.r.t. y)

$$\frac{1}{\mu}\cdot\frac{\partial p}{\partial x}\cdot y + c_1 = \frac{du}{dy}$$

integrating again, (w.r.t. y)

$$\frac{1}{\mu}\cdot\frac{\partial p}{\partial x}\cdot\frac{y^2}{2} + c_1 y + c_2 = u$$

now, lets look at the boundary conditions (1, 2, 3)

(1) at $y=0$, $u=0$

$$0 + 0 + c_2 = 0 \quad \therefore c_2 = 0$$

u= assuming parabolic flowfield, height of pipe, h

(3) also, at $y=h$, $u=0$

$$\frac{1}{\mu}\cdot\frac{\partial p}{\partial x}\cdot\frac{h^2}{2} + c_1 h = 0$$

$$c_1 = -\frac{1}{\mu}\cdot\frac{\partial p}{\partial x}\cdot\frac{h}{2}$$

$$u = \frac{1}{\mu}\cdot\frac{\partial p}{\partial x}\cdot\frac{y^2}{2} - \frac{1}{\mu}\cdot\frac{\partial p}{\partial x}\cdot\frac{h}{2}\cdot y$$

$$= \frac{\partial p}{\partial x}\cdot\frac{1}{2\mu}(y^2 - hy) \quad (A)$$

assuming a parabolic flow field like the one sketched

h

(2) at $y = \frac{h}{2}$ (where, h is the height of the pipe)

$u = u_{max}$

between (// //) → wrong stuff

$$\text{so, } \frac{1}{\mu}\cdot\frac{\partial p}{\partial x}\cdot\frac{h^2}{8} + c_1\cdot\frac{h}{2} = u_{max}$$

$$c_1\cdot\frac{h}{2} = u_{max} - \frac{h^2}{8\mu}\cdot\frac{\partial p}{\partial x}$$

$$c_1 = \frac{2u_{max}}{h} - \frac{h}{4\mu}\cdot\frac{\partial p}{\partial x}$$

$$\therefore u = \frac{1}{\mu}\cdot\frac{\partial p}{\partial x}\cdot\frac{y^2}{2} + \left(\frac{2u_{max}}{h} - \frac{h}{4\mu}\cdot\frac{\partial p}{\partial x}\right)y$$

$\boxed{\text{at } h=0 \text{ this term blows up to infinity; so that can not be right}}$

as, $v = 0 = w$, $\vec{V} = u\hat{i}$

$$\therefore \vec{V}(y) = \left[\frac{1}{\mu}\cdot\frac{\partial p}{\partial x}\cdot\frac{y^2}{2} + \left(\frac{2u_{max}}{h} - \frac{h}{4\mu}\cdot\frac{\partial p}{\partial x}\right)\cdot y\right]\hat{i}$$

because, fully developed and 2-D

@ $y=0$, $u=0$; $y=h$, $u=0$ } so, this works; also, the equation (y^2-hy) looks like a parabolic eqn, so that's good

(A) $u = \frac{\partial p}{\partial x}\cdot\frac{1}{2\mu}(y^2 - hy)$ $\quad v = 0 = w$, $\vec{V} = u\hat{i}$ $\quad$ h = ~~diameter~~ internal diameter or height of the pipe

FINAL ANSWER

$$\boxed{\therefore \vec{V}(y) = \left[\frac{\partial p}{\partial x}\cdot\frac{1}{2\mu}(y^2 - hy)\right]\hat{i}}$$

[here, $\frac{\partial p}{\partial x}$ can be replaced as $\pm(P_2 - P_1)$ depending on the direction of the flow field]

because it does not change w.r.t. to x and z because fully developed and 2-D flow.

CHEE 314: Fluid Mechanics **Quiz #3 (2015)**

STUDENT NAME: ________________________ **STUDENT #:** _______________

Q1. When a cylindrical tank is filled with water, the bottom of the tank typically deflects under the weight of the water inside. The deflection at the center of the tank (δ) is a function of the tank diameter *D*, the height of the water *h*, the thickness of the tank bottom *d*, the specific weight of the water γ, and the modulus of elasticity of the tank material *E*. Determine the functional relationship among these parameters using dimensionless groups.

[6 points]

6 variables : $\delta, D, h, d, \gamma, E$; $n = 6$

	δ	D	h	d	γ	E
M	0	0	0	0	1	1
L	1	1	1	1	-2	-1
T	0	0	0	0	-2	-2

Rank of the matrix = 2

Repeating variables = 2 , γ and E

Pi group = 6 − 2 = 4

$\Pi_1 = \delta \gamma^a E^b$, Thus, $L \times M^a L^{-2a} T^{-2a} \times M^b L^{-b} T^{-2b} = M^0 L^0 T^0$

M: $a + b = 0$ L: $1 - 2a - b = 0$ T: $a + b = 0$

$\Rightarrow a = 1, \quad b = -1$

$\Pi_1 = \dfrac{\delta\gamma}{E}$, similarly, $\Pi_2 = \dfrac{D\gamma}{E}$, $\Pi_3 \dfrac{h\gamma}{E}$ and $\Pi_4 = \dfrac{d\gamma}{E}$

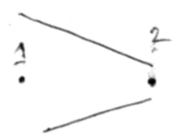
$\Pi_1 = f(\Pi_2, \Pi_3, \Pi_4)$

Q2. A fire nozzle is coupled to the end of a hose with inside diameter D = 4 inches. The nozzle is contoured smoothly and has an outlet diameter d = 0.5 inches. The design inlet pressure for the nozzle is 100 psi (gage). Evaluate the maximum water flow rate the nozzle could deliver, assuming frictionless flow.

[4 points]

$P_2 = P_{atm} = 0$ (gage)

$Z_1 = Z_2$



$$\frac{P_1}{\rho} + \frac{v_1^2}{2} + g\cancel{Z_1} = \cancel{\frac{P_2}{\rho}} + \frac{v_2^2}{2} + g\cancel{Z_2}$$

$$\frac{P_1}{\rho} = \frac{1}{2}\left(v_2^2 - v_1^2\right)$$

$$P_1 = \frac{\rho}{2}\left(v_2^2 - v_1^2\right)$$

$$Q = v_1 A_1 = v_2 A_2$$

$$\Rightarrow v_1 D_1^2 = v_2 D_2^2$$

$$v_1 = v_2 \frac{D_2^2}{D_1^2} = v_2 \frac{0.5^2}{4^2} = \frac{v_2}{64}$$

Now,

$$P_1 = \frac{\rho}{2}\left(v_2^2 - \frac{v_2^2}{64^2}\right)$$

$P_1 = 100\ \text{psi} = 100 \times 6.89\ \text{kPa} = 689 \times 10^3\ \text{Pa}$

$\rho = 1000\ \text{kg/m}^3$

$$\frac{2P_1}{\rho} = v_2^2\left(1 - \frac{1}{64^2}\right)$$

$$\boxed{v_2 = 37.12\ \text{m/s}}$$

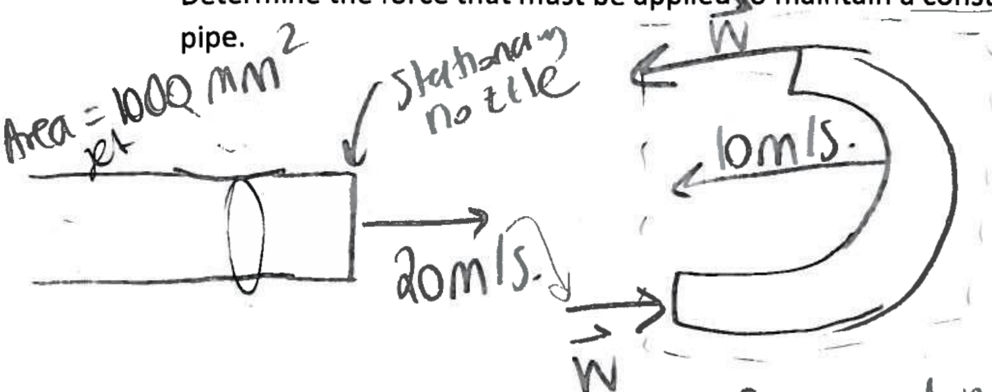
$$Q = v_2 A_2 = \boxed{4.7 \times 10^{-3}\ \text{m}^3/\text{s}}$$

$$\vec{V} = \vec{V}_{CV} + \vec{W}$$

Q1. A jet of water is launched horizontally from a stationary nozzle, and enters a curved pipe that deflects the water by 180 °. The jet area is 1000 mm^2 and its speed relative to the stationary nozzle is 20 m/s. The curved pipe moves towards the nozzle at 10 m/s. Determine the force that must be applied to maintain a constant speed of the curved pipe.

[5 points]

5



F to have no acceleration of curved pipe?

$$\Sigma F_x = \underbrace{\frac{\partial}{\partial t}\int_{CV} \rho \vec{V}_{CV}\, dVol}_{\text{CV not accelerating}} + \underbrace{\frac{\partial}{\partial t}\int_{CV} \rho \vec{W}\, dVol}_{\text{Steady flow}} + \underbrace{\int_{CS} V_{CV}\, \rho (\vec{W}\cdot\hat{n})\, dS}_{\text{con. of mass}} + \int_{CS} \vec{W} \rho (\vec{W}\cdot\hat{n})$$

① We need to find $\vec{W}$

$$\vec{V} = \vec{V}_{CV} + \vec{W}$$

$$20 = -10 + \vec{W} \qquad \boxed{\vec{W} = 30 \text{ m/s}}$$

$$F = \underbrace{-\vec{W}^2 \rho A}_{\text{goes in}} + \underbrace{-W^2 \rho A}_{\text{comes out}}$$

$(-\vec{W})(\vec{W})$ $\quad$ $(\vec{W})(-\vec{W})$

From cons. of Mass $A_1 V_1 = A_2 V_2$ of the curved pipe.

$$F = -2\vec{W}^2 \rho A$$

$$F = -2(30)^2\, 1000\, (1000)\cdot 10^{-3} \cdot 10^{-3}$$

$$\boxed{F = -1800 \text{ N}}$$

force on the CV

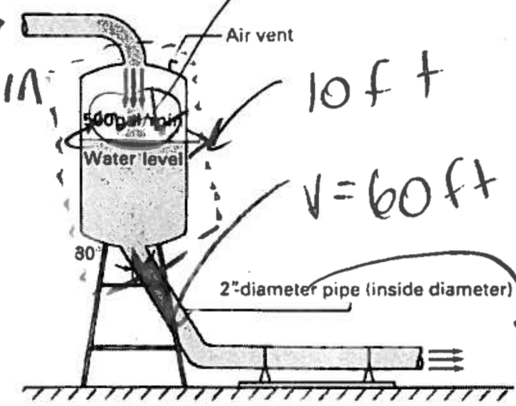
$\frac{500 \text{ gal}}{\text{min}} \cdot .6$ $\quad 9.1138 \frac{ft^3}{s}$ $\quad 66.8 \frac{ft^3}{min}$

Q2. Water is being added to a storage tank at the rate of 65 cubic feet/min. Water also flows out of the bottom through a 2.0 in (inside diameter) pipe with an average velocity of 60 ft/sec. The inside diameter of the storage tank is 10.0 ft. Find the rate at which the water level is rising or falling.

I'll consider this as what to use.

[5 points]

$65 ft^3/min$



10 ft

V = 60 ft/s.

$2in \cdot \frac{1ft}{12in} = 0.1667 ft$

$\frac{dy}{dt} = ?$

conservation of Mass

$$0 = \frac{\partial}{\partial t}\int_{cv} \rho \, dVol + \int \rho \vec{V} \cdot \hat{n} \, ds$$

$$0 = \frac{\partial}{\partial t}\int \cancel{\rho} \frac{\pi D^2}{4} \cdot \overset{\text{height}}{Y} - \overbrace{\cancel{\rho} V_{in} A_{in}}^{Q_{in}} + \cancel{\rho} V_{out} A_{out}$$

Assume: Incompressible

$$0 = \frac{\partial y}{\partial t}\frac{\pi D^2}{4} - Q_{in} + V_{out} A_{out}$$

$$0 = \frac{\partial y}{dt}\frac{\pi \, 10ft^2}{4} - \frac{65 ft^3}{min} + \frac{60 ft}{s} \cdot \frac{60 s}{1 min} \cdot \frac{\pi (0.16667 ft)^2}{4}$$

$$-\frac{\partial y}{dt} \cdot \frac{\pi 10^2}{4} = \frac{13.5 \, ft^3}{min}$$

$$\boxed{\frac{dy}{dt} = -0.1727 \, ft/min}$$

* Assume ρ doesn't change w/ temp *

$V = \frac{m}{\rho}$

Q3. You are enjoying a glass of chilled orange juice (SG = 1.1), on a hot summer day. Several ice cubes (made from water) float on the liquid surface. As the ice cubes melt, will the level of the liquid in the glass go UP, DOWN, or STAY THE SAME? Explain.

[5 points]

Before

② After melting

not displacing OJ anymore. It's going to be sitting on the top as shown above

Before melting.

① $\rho_{orange\ juice} V_{disp} \cancel{g} = m_{ice\ cubes} \cancel{g}$

$V_{disp} = \frac{m_{ice\ cubes}}{\rho_{orange\ juice}}$

$\rho_w V_{disp} \cancel{g} = (V_{ice})(\rho_w) \cancel{g}$

$\boxed{V_{disp} = \frac{(V_{ice\ cube}) \cancel{\rho_w}}{\cancel{\rho_w}}}$

$\boxed{V_{disp\ by\ ice\ cube} = \frac{(V_{ice\ cube})(\rho_w)}{\rho_{OJ}}}$

SG = 1

SG = 1.1

So...

$\boxed{V_{disp_1} = \frac{(V_{ice\ cube})_{total}}{1.1}}$

m/ρ_{ice}

m/ρ_{ice}

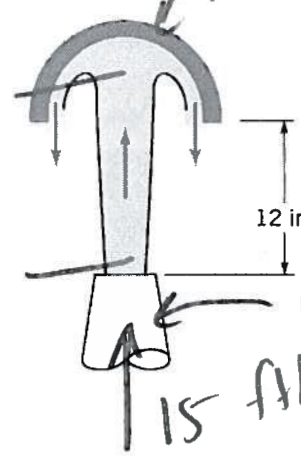
$\boxed{V_{disp_2} = (V_{ice\ cube})_{total}}$

therefore level of liquid will Increase

⟹ Mass conserved
∴ & assume
⟹ ρ doesn't change
⟹ Assume volume won't change from melting

Q4. A vertical jet of water having a nozzle exit velocity of 15 ft/s with a diameter of 1 in. suspends a hollow metal hemisphere as shown in the figure. If the hemisphere is balanced at an elevation of 12 in., determine its' mass. State any assumptions made.

[10 points]



① Conservation of Momentum

Assume: Steady state

$$\Sigma F_y = \int_{cs} \rho \vec{V}_y \vec{V} \cdot \hat{n} \, dS$$

$$-mg = \underbrace{-\rho V_{in}^2 A_{in}}_{\text{entering}} - \rho \overset{\text{pointing downwards}}{V_{out}^2} A_{out}$$ ③

$$-mg = -\rho_{water}\left(\left(\frac{15 ft}{s}\right)^2 \cdot \underbrace{\frac{(0.0833 ft)^2}{4}\pi}_{5.4537 \cdot 10^{-3}} + (12.67)^2 (6.456 \cdot 10^{-3}\right)$$

$$-mg = -\rho_w \left(\frac{2.26 ft^4}{s^2}\right)$$

$$mg = \frac{1000 kg}{m^3} \cdot \frac{2.26 ft^4}{s^2}$$

$$\boxed{m = 1.99 kg = 0.136 \text{ slug}}$$ 1.82

② need to find what V_{out} is, use Bernoulli

$$\cancel{\frac{P}{\rho}}^{0} + \frac{V_1^2}{2} + \cancel{gz}^{0} = \cancel{\frac{P}{\rho}}^{0} + \frac{V_2^2}{2} + gz$$ ②

both exposed to atmospheric pressure.

$$\frac{\pi (0.0833)^2}{4}$$

$$\left(\frac{15 ft}{s}\right)^2 \cdot \frac{1}{2} = \frac{V_2^2}{2} + \frac{32.174 ft}{s^2} \cdot 1 ft$$

$$V_2 = 12.67 \text{ ft/s}.$$

③ Conservation ~~of mass~~ to get A_{out} ②

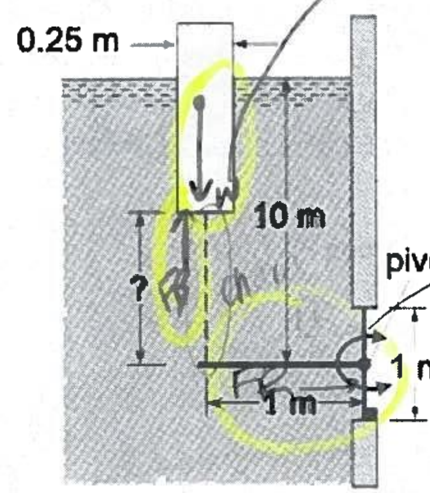
$$V_1 A_1 = V_2 A_2 \rightarrow 12.67$$

$$(15)\pi \, ^2 = 6.456 \cdot 10^{-3} \, ft^2$$

10.

Q5. A gate with a circular cross section is held closed by a lever ~~1 m long~~ attached to a buoyant cylinder. The cylinder is 25 cm in diameter and weighs 200 N. The gate is attached by a rigid connection to a horizontal shaft so it can pivot about its center (as shown). The liquid is water, at a density of 1000 kg/m³. You may assume the chain and lever have negligible mass and buoyancy. What length chain is needed to get the gate to open when the water level is above 10 m?

[10 points]



hinge is @ gates center

① Find force of fluid on gate ✓1.0

$F_R = \rho g \sin\theta\, y_c A$ → circle radius $\frac{1}{2}$

$= \rho g (10)(\pi(\frac{1}{2})^2)$ → distance of y_c from surface.

$\boxed{= 7.7047 \cdot 10^4 \text{ N}}$

② Find y' ✓ I_{xcxc}. $\frac{1}{4}\pi r^4$

$y' = y_c + \frac{\rho g \sin\theta}{\rho g \sin\theta\, y_c A}$

$y' = 10 + \frac{1}{10 \cdot \pi (0.5)^2} \frac{1}{4}\pi(0.5)^4$ ✓1.0

$\boxed{y' = 10.00625 \text{ m}}$ from surface.

distance from hinge

$\boxed{= 0.00625 \text{ m}}$ ✓

③ For the gate we have $W = 200N$ @ a perp distance of 1 m from hige

④ Buoyant force 481.54h ✓1

$F_B = \rho g V_{disp} = 1000 g \left(\frac{\pi 0.25^2}{4}\right) h$

what h do we need to have disp under

⑥ $10 - 1.415 =$ ✓ $\boxed{8.535 \text{ m}}$

⑤ ΣM about hinge $= 0$ ✓1.0

4.0

$(F_R)(0.00625) + (W)(1m)$ ✓

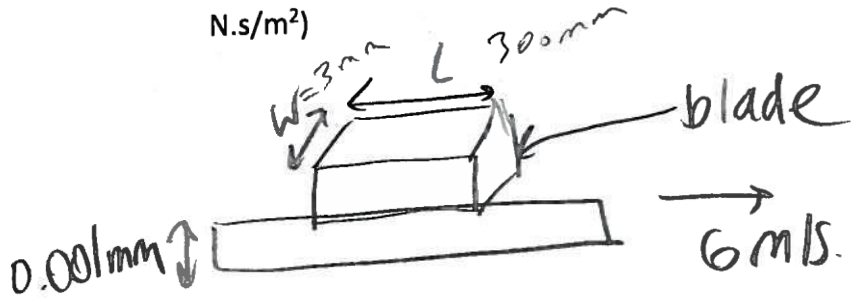
$-(1)(1000 g \frac{\pi 0.25^2}{4} h) = 0$ ✓

$\boxed{h = 1.415 \text{ m}}$

2.0

Q6. An ice skater glides on one skate at speed V = 6 m/s. Her weight is supported by a thin film of liquid water melted from the ice by the pressure of the skate blade. Assume the blade is L = 300 mm long and w = 3 mm wide, and that the water film is h = 0.001 mm thick. Estimate the deceleration of a skater who weighs 60 kg as a result of viscous shear in the water film, if end effects on the skate are neglected. ($\mu_{water} = 1.76 \times 10^{-3}$ N.s/m^2)

5 [5 points]



Assume $Re < 2300$ so laminar flow.

$$\tau = \mu \frac{du}{dy}$$

$$\frac{F}{A} = \mu \frac{du}{dy}$$

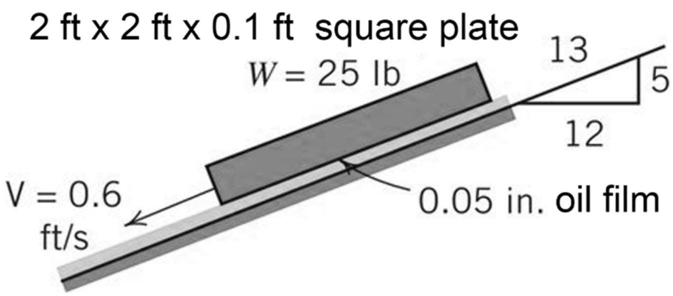
$$\frac{ma}{A_{contact}} = \mu \frac{du}{dy}$$

$$\frac{(60)(a)}{(0.3)(3\cdot 10^{-3})} = \frac{1.76\cdot 10^{-3}(0-6)}{0.001\cdot 10^{-3}}$$

$$\boxed{a = -0.1584 \ \frac{m}{s^2}}$$

STUDENT NAME: _________________________ STUDENT #: ________________

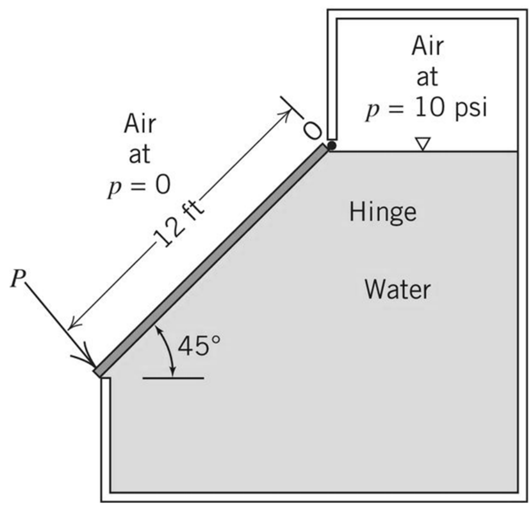
<u>Q1.</u> Calculate the approximate viscosity of the oil. State assumptions, if any.



<u>[4 points]</u>

Q2. Calculate the minimum force *P* necessary to hole a uniform 12 ft square gate weighing 500 lb closed on a tank of water under a pressure of 10 psi. State assumptions made, if any.

[6 points]

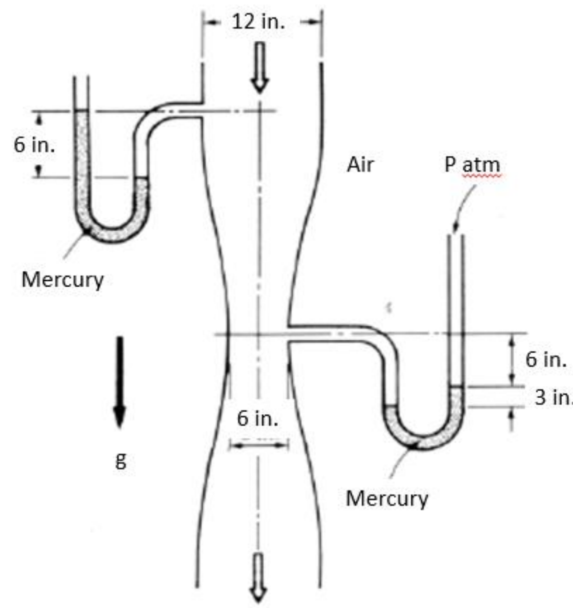


CHEE 314: Fluid Mechanics **Quiz #2 (2017)**

STUDENT NAME: __________________________ STUDENT #: _________________

1. Water flows vertically downward through a Venturi meter as shown. The dimensions of the Venturi and the levels in the mercury manometers are as shown in the diagram. The inlets to the manometers are 2 feet apart. Estimate the flow rate through the Venturi meter in ft^3/sec. The density of water is 62.4 lbm/ft^3 and the density of mercury is 13.6 times greater than that of water. Neglect any frictional losses.

   [5 points]



STUDENT NAME: ________________________ STUDENT #: _______________

<u>Q2.</u> Water (μ= 1 mPa.s, ρ = 1000 kg/m^3) flows between two large, horizontal parallel plates, driven by a pressure gradient applied along the x-axis. The plates are spaced 0.5 cm apart. A U-tube manometer filled with mercury (ρ = 13.6 SG) connects two points along the bottom, 15 cm apart, and shows a height difference of 3mm. You may assume laminar flow.

a) Draw a diagram and indicate which arm of the manometer will have a higher fluid level. **[1 point]**

b) Determine the flow profile and calculate the maximum velocity of the fluid between the two plates. State any assumptions made.

[4 points]

STUDENT NAME: ________________________ STUDENT #: ________________

Q1. Air at standard temperature and pressure flows through a horizontal, galvanized iron pipe (ε = 0.0005 ft) at a rate of 2.0 ft^3/s. The pressure drop is to be no more than 0.005 psi/ft of pipe. Determine the minimum pipe diameter. Make assumptions as needed.

Note: ρ = 0.00238 $slugs/ft^3$, $\mu = 3.74 \times 10^{-7}$ $lb.s/ft^2$ **[5 points]**

STUDENT NAME: ________________________ STUDENT #: _______________

<u>Q2.</u> The aerodynamic behaviour of a flying insect is to be investigated in a wind tunnel using a 10x larger robotic insect model with controllable flapping wings. If the real insect flaps its wings 60 times per second when flying at 1.5 m/s, determine the wind tunnel air speed and flapping rate of the robot required for model-prototype similarity. Hint: consider viscosity ν of air = 1.50×10^{-5} m^2/s.

[5 points]

Department of Chemical Engineering
McGill University
CHEE 314 Fluid Dynamics

Midterm Fall 2017

STUDENT NAME: ______________________

STUDENT NUMBER: ______________________

Regular Instructions:

Time: 1 hour, 50 minutes (11:35 AM to 13:25 PM)

There are **SIX** questions; the weight of which is indicated on the question. (Total = 30 points)

Allowable aids: Calculator, dictionary, translators, up to 20 pages of *hand-written* notes.

REALLY IMPORTANT instructions:

Quickly scan through all the problems before starting.

Then: Read the problem a second time carefully. Think about what it is asking.

Then think about it again.

If you don't know where to start, go back to the problem solving framework:

1) draw & state assumptions; 2) define physics; 3) where? ; 4) solve;
If appropriate, 5) check units + assumptions

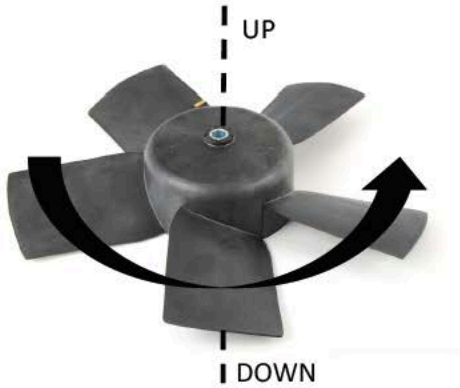
Finally:

Relax. Don't panic. Breathe. You've got this.

Do not write on this page – administrative use only.

Q 1 / 5		
Q 2 / 5		
Q 3 / 5		
Q 4 / 5		
Q 5 / 5		
Q6 / 5		
Total / 30		

<u>Q1 A.</u> A fan has blades angled as shown. When it spins in the labelled direction, will the fan blow air up or down?



[2 points]

ANSWER (CIRCLE THE RIGHT ONE):

UP / DOWN

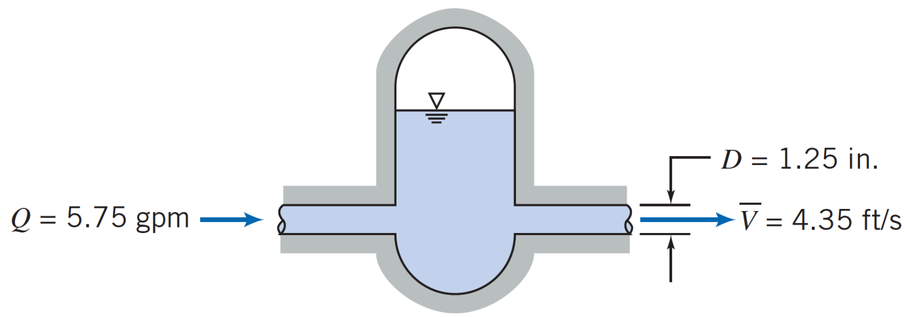
<u>Q1 B</u>. A helicopter maintains stationary position 10 m above sea level. With all other conditions being equal, would the blades need to spin FASTER, SLOWER, or AT THE SAME RATE to maintain stationary position at 100 m above sea level?

[3 points]

ANSWER:

<u>Briefly</u> explain your answer:

<u>Q2.</u> A hydrodynamic accumulator is designed to reduce pressure pulsations in a machine tool hydraulic system. For the instant shown, determine the rate at which the accumulator gains or loses hydraulic oil. (ρ_{oil} = 0.9 SG)

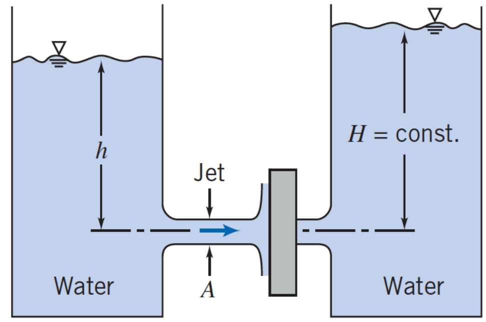


[5 points]

<u>Q3</u>. A bucket floats, open side up, on the ocean (consider ρ_{H20} = 1000kg/m^3). The bucket is 20 cm in diameter and 10 cm tall. Starting from an initial empty state, the bucket floats on the water surface at a depth of 5 cm. If the bucket starts to leak, and takes on water at a rate of 10mL/min, how long will it take to sink to 8cm?

[5 points]

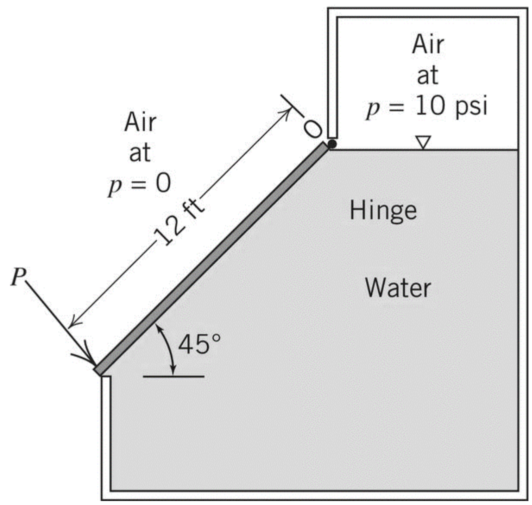
Q4. Two large tanks containing water have small smoothly contoured orifices of equal area. A jet of liquid issues from the left tank. Assume the flow is uniform and unaffected by friction. The jet impinges on a vertical flat plate covering the opening of the right tank. Determine the minimum value for the height, *h*, required to keep the plate in place over the opening of the right tank.



[5 points]

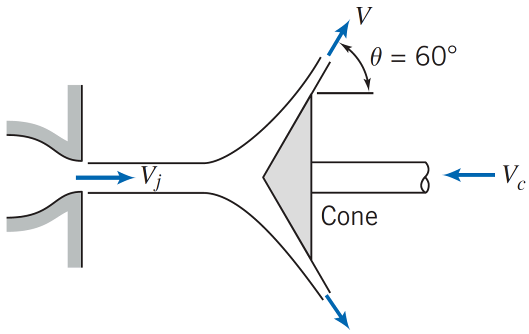
<u>Q5.</u> Calculate the minimum force *P* necessary to hold a uniform 14 ft square gate weighing 500 lb closed on a tank of water under a pressure of 10 psi. State assumptions made, if any. **<u>Show all work.</u>**

[5 points]



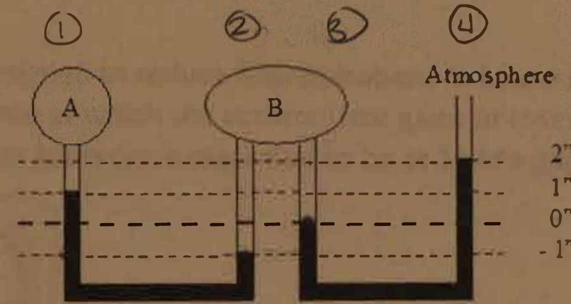
<u>Q6</u>. Water, in a 4-in. diameter jet with speed of 100 ft/s to the right, is deflected by a cone that moves to the left at 45 ft/s. Determine (a) the thickness of the jet sheet at a radius of 9 in. and (b) the external horizontal force needed to move the cone.

[5 points]



Statics

Q1 A. Two tanks filled with air are connected by water-filled manometers as shown. The water levels are as shown. Choose the correct answer for the gage pressure for Tank A.



$P_3 = P_{atm} + 2in$

___ Pa = + 3" water

✓ Pa = + 1" water

___ Pa = + 0" water

___ Pa = - 1" water

___ Pa = - 3" water

$P_2 = P_1 + 2in$ $\quad P_2 = P_3 + 1in$

$P_1 = P_2 - 2i$

$P_1 = P_3 - 1in$

$P_1 = P_{atm} + 1in$

[2 points]

0/2

Momentum

Q1 B. A helicopter maintains a stationary position 10 m above sea level. If any differences in air density are neglected, would the blades need to spin FASTER, SLOWER, or AT THE SAME RATE to maintain stationary position at 100 m above sea level?

[2 points]

2/2

ANSWER: the same

Briefly explain your answer:

mg must be balanced by forces in the y

→ gravity does not change with height

$\sum F_y = \cancel{\frac{d}{dt}\int_{cv} \rho\, dVol} + \int_{cs} \rho u_y \vec{v}\cdot\vec{n}\, ds$

steady state

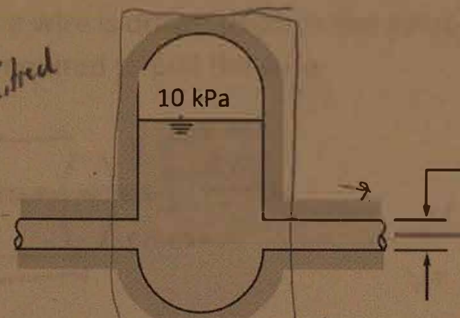
$\sum F_y = \int \rho u_y v n ds$

Δ height has no effect on ρ or v, so no changes necessary (constant area)

Cons. of Mass

Q2. A hydrodynamic accumulator is designed to reduce flow pulsations in a flow line. For the instant shown, determine the rate at which the accumulator gains or loses hydraulic oil. (ρ_{oil} = 0.9 SG; accumulator is pressure regulated to be at 10 kPa gage)

what goes in must come out
→ pressure doesn't really need to be used because velocity is specified



≈ 3.175×10^{-2} m → $R = 1.5875\times10^{-2}$ m

≈ 1.3259 m/s

[5 points]

$$5.75 \text{ gpm} \times 6.309\times10^{-5} = 3.628\times10^{-4}\ \frac{m^3}{s}$$

$Q_{in} = Q_{out}$ (conservation of mass)

$$0 = \frac{d}{dt}\int_{cv}\rho\, dVol + \int_{cs}\rho\vec{v}\cdot\hat{n}\, dS$$

$$0 = \frac{dV}{dt} - 3.628\times10^{-4}\frac{m^3}{s} + \pi R^2 V$$

$$0 = \frac{dV}{dt} - 3.628\times10^{-4}\frac{m^3}{s} + \pi(1.5875\times10^{-2}\,m)^2(1.3259\ \tfrac{m}{s})$$

$\rho = 1000\ \frac{kg}{m^3}$

$$3.628\times10^{-4} = \left(\frac{dV}{dt} + 1.04976\times10^{-3}\right)$$

$$\frac{dV}{dt} = -6.869\times10^{-4}\ \frac{m^3}{s}$$

→ volume is decreasing by $6.869\times10^{-4}\ \frac{m^3}{s}$

-2

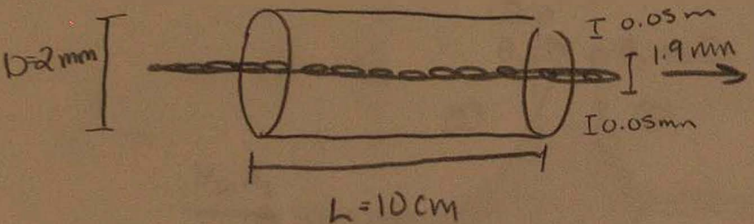
2

1

Shear Stress

Q3. A copper wire is to be coated with paint for insulation by drawing it through a circular tube of 2.0 mm diameter, and length of 10 cm. The wire diameter is 1.9mm and is centered in the tube. The paint (μ = 10 Pa.s) completely fills the space between the wire and the tube. The wire is drawn through the paint-filled tube at a speed of 50 m/s. Determine the force required to pull the wire.

[5 points]



$\mu = 10\ Pa \cdot s$

$V = 50\ m/s$

$$\tau = \frac{F}{A} = \mu \frac{du}{dy}$$

$$F = A \cdot \mu \cdot \frac{du}{dy}$$

surface area

$$= \pi d h \cdot \mu \cdot \frac{du}{dy}$$

$$= \pi \left(\frac{1.9}{1000} m\right)\left(\frac{10}{100} m\right) \cdot 10\ Pa\ s \cdot \frac{50\ m/s}{\frac{0.05}{1000}}$$

$$= \boxed{5969\ N}$$

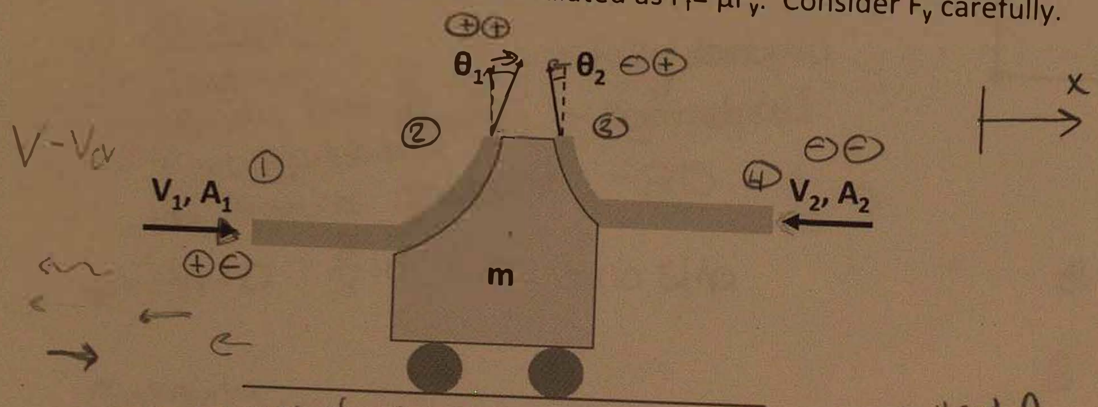
Consv of Momentum

12/5

Q4. A cart with wheels is being sprayed by two 0.05 m^2 jets that are being deflected off the surface at speeds of V_1=10 m/s and V_2=30 m/s and at angles of θ_1=55° and θ_2=40°, respectively. The cart has a mass of m=10kg and a coefficient of friction μ=0.02 with the ground beneath it. What is the terminal velocity and the direction of the cart?

Hint: Friction force can be calculated as $F_f = \mu F_y$. Consider F_y carefully.

$V - V_{cv}$



[5 points]

→ neglect mass of jet water in cv
→ assume liquid = water
→ assume constant V and A in jets
→ Find out which way cart is moving
→ x is positive to the right

$$\sum F_x = \cancel{\frac{d}{dt}\int_{cv} \rho \, dVol}^{\,0} + \int_{cs} \rho v_x \vec{v}\cdot\hat{n}\, ds$$

$$= \rho\left(V_1(-V_1)A_1 + V_1 \sin\theta_1 \cdot V_1 A_1 + (-V_2)\sin\theta_2 \cdot V_2 \cdot A_2 + (-V_2)(-V_2)A_2\right)$$

use $\vec{w}$ b/c moving cv, not $\vec{v}$ ~~to~~

$$= 1000(-100 \cdot 0.05 + 100 \cdot \sin(55) \cdot 0.05 - 30^2 \sin(40) \cdot 0.05 + 30^2 \cdot 0.05)$$

$$= 1000 \cdot 0.05(-100 + 100\sin(55) - 30^2 \sin(40) + 30^2)$$

$$= 1000 \cdot 0.05(303.4)$$

$$= 15\,170.3 \text{ N}$$ ← force needed to balance is to the right, so cart is moving to the $\boxed{\text{left}}$

$w_1 = 1$

$F_f = \mu \cdot m \cdot g$
$= 0.02 \cdot 10 \cdot 9.81$
$= 1.962$ N
to the right

+0.5/1 forgot F_y due to water

→ want some w where $\sum F_x = 1.962$ N
→ assume same w for all

+0.5/1 for $\vec{w}$

$$= 1000 \cdot 0.05(-(10 - V_{cv})^2 + (10 - V_{cv})^2 \sin(55) - (30 + V_{cv})^2 \sin(40) + (30 + V_{cv})^2) \;?$$

$$1.962 \text{ N} = 1000 \cdot 0.05 \, w^2(-1 + \sin(55) - \sin(40) + 1)$$

+ 1/1

$$w = 0.471 \text{ m/s}$$

← assume same w for all

$w_1 \neq w_2$

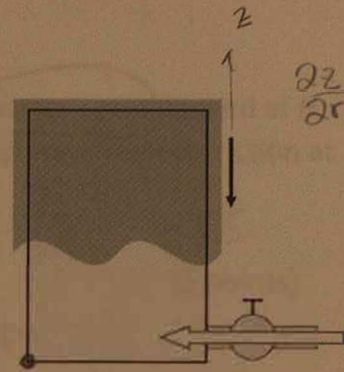
ma = 15 170 N

$a = 1517 \frac{m}{s^2}$

V

N-S

Q5. When the supply valve is opened, a very viscous Newtonian liquid flows into a very large cylindrical tank, which overflows. Determine the velocity profile of the liquid on the outside of the cylinder as it slowly spills down the sides. State any assumptions made.



$\frac{\partial z}{\partial r}$

1/5

[5 points]

Assumptions
- Newtonian
- Steady flow $\frac{\partial}{\partial t} = 0$
- Incompressible
- No Pressure gradient
- fully developed
- symmetrical
- 2D $\frac{\partial}{\partial \theta} = 0$
- no slip

$V_\theta = V_r = 0 \quad \frac{\partial}{\partial r} = 0$

@ $r=0$ $v=0$ (no slip)

@ $r=R$ $v=V_0$ – no

↳ outside velocity

Continuity

$$\cancel{\frac{1}{r}\frac{\partial(rV_r)}{\partial r}} + \cancel{\frac{1}{r}\frac{\partial V_\theta}{\partial \theta}} + \frac{\partial V_z}{\partial z} = 0$$

$$\frac{\partial V_z}{\partial z} = 0 \qquad \text{show } V_r = 0$$

$\frac{\partial P}{\partial t} = ?$

N-S

$$z: \rho\left(\cancel{\frac{\partial V_z}{\partial t}} + \cancel{V_r\frac{\partial V_z}{\partial r}} + \cancel{\frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta}} + \cancel{V_z\frac{\partial V}{\partial z}}\right) = \cancel{-\frac{\partial P}{\partial z}} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2}} + \cancel{\frac{\partial^2 V_z}{\partial z^2}}\right] + \rho g_z$$

steady $\quad V_r = 0 \quad V_\theta = 0 \quad \frac{\partial V_z}{\partial z} = 0 \quad$ no P gradient $\quad \frac{\partial}{\partial \theta} = 0 \quad \frac{\partial V_z}{\partial z^2} = 0$

$$\rho g_z = \mu \cdot \frac{1}{r}\frac{\partial}{\partial r} r\frac{\partial V_z}{\partial r}$$

$$\rho g_z = \frac{\mu}{r}\cdot\frac{\partial}{\partial r} r\frac{\partial V_z}{\partial r}$$

$$\frac{\rho g_z r}{\mu} = \frac{d}{\partial r} r\frac{\partial V_z}{\partial r}$$

$$\frac{1}{2}\frac{\rho g_z r^2}{\mu} + C_1 = r\frac{\partial V_z}{\partial r}$$

$$\frac{1}{2}\frac{\rho g_z r}{\mu} + \frac{C_1}{r}\, dr = \partial V$$

$$V = \frac{1}{4}\frac{\rho g_z r^2}{\mu} + C_1 \ln|r| + C_2$$

(+1)

Boundary conds.

$$0 = C_1 \ln|0| + C_2$$

$$C_2 = 0$$

$$V = \frac{\rho g_z r^2}{4\mu} + C_1 \ln|r|$$

$$V_0 = \frac{\rho g R^2}{4\mu} + C_1 \ln R$$

$$\frac{4 V_0 \mu}{\rho g R^2}\cdot\frac{1}{\ln R} = C_1$$

$$\boxed{V = \frac{1}{4}\frac{\rho g r^2}{\mu} + \frac{4 V_0 \mu}{\rho g R^2}\frac{\ln r}{\ln R}}$$

X

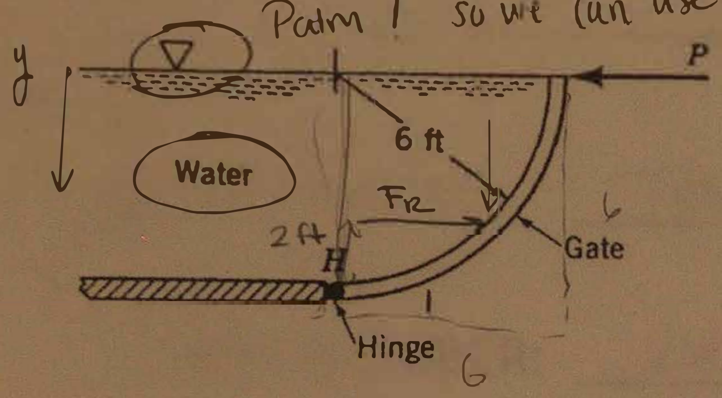
Statics

Q6. The 18-ft-long lightweight gate shown in the figure is a quarter circle, hinged at *H*. Determine the horizontal force *P*, required to hold the gate in place. Neglect friction at the hinge and the weight of the gate.

[5 points]

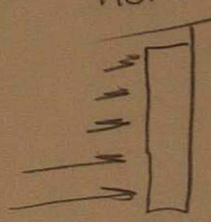
5/5

Patm ! so we can use $y' = y_c + \frac{I_{\hat{x}\hat{x}}}{y_c A}$



$w = 18$ ft

Horizontal



$F_R = \rho g y_c A$ @ y'

$F_R = (1.94)(32.2) \cdot \frac{6}{2} \cdot 6 \cdot 18$

$F_R = 20{,}239.6$ lbf (+1)

$y_c = \frac{h}{2}$

$y' = y_c + \frac{\frac{1}{12} bh^3}{y_c \cdot A}$

$= \frac{h}{2} + \frac{\frac{1}{12}(18)(6)^3}{\frac{h}{2} \cdot 6 \cdot 18}$

$= 3 + 1$

$= 4$ ft

Vertical

$F_v = \rho V g$

$V = \frac{1}{4} \pi r^2 \cdot W$

$F_v = \rho \cdot \frac{1}{4} \cdot \pi r^2 \cdot W \cdot g$

$= 1.94 \cdot 32.2 \cdot \frac{1}{4} \cdot \pi \cdot 6^2 \cdot 18$

$= 31{,}792.3$ lbf (+1)

acts @ $\bar{x}$ (+1)

$\bar{x} = \frac{4r}{3\pi} = \frac{4 \cdot 6}{3\pi} = 2.546$ ft

$\Sigma M = 0$ (+1)

$(6)(P) = (2) F_R + 2.546 \cdot F_V$

$P = \frac{2(20{,}239.6) + 2.546(31{,}792.3)}{6}$

$\boxed{P = 20\,237 \text{ lbf}}$ (+1)

6.5+

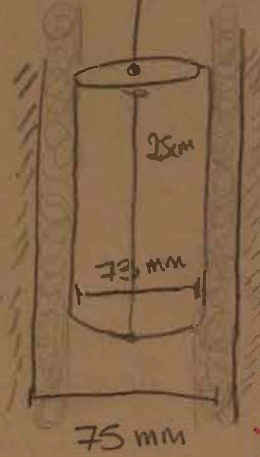
CHEE 314: Fluid Mechanics **Quiz #1 (2018)**

STUDENT NAME: ____ **STUDENT #:** _

Q1. An aluminum cylinder (SG = 2.64, diameter 73mm, length 25cm) is inside a vertical steel tube with an inner diameter of 75mm. The inner wall of the tube is coated with a thin film of motor oil ($\mu = 2 \times 10^{-1}$ N-s/m^2). The cylinder weighs 3 kg and is attached to a support cable, which is used to pull the cylinder upwards at a constant speed of 3 mm/s. What is the tension applied on the support cord? State any assumptions made.

assume no slip

[4 points]



$\mu = 2 \times 10^{-1} \frac{N \cdot s}{m^2}$

$m = 3kg$

$u = 3$ mm/s

thickness of oil = 1 mm on each side

$\tau = \frac{F_1}{A} = \mu \frac{du}{dy}$

$\frac{F}{A} = (2 \times 10^{-1} \frac{N \cdot s}{m^2}) \cdot \frac{.003 \text{ m/s}}{.001 \text{ m}}$

$\frac{F}{A} = 0.6$

x 2 for above + below

$\frac{F}{A} = 1.2 \frac{N}{m^2}$

A = SA of ~~sp~~ cylinder

~~A=~~ $C = \pi d = 2\pi r$

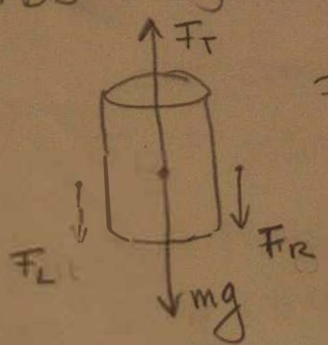
$SA = 2\pi r h$

$= 2\pi(.0365 \text{ m})(.25 \text{ m})$

$= 0.0573 \text{ m}^2$

$F = 0.0688$ N

FBD on cylinder



$F_L + F_R$ caused by shear stress

Force Balance:

$F_T = mg + \underbrace{F_L + F_R}_{F}$

$mg = 3kg(9.81 \frac{m}{s^2}) = 29.43$ N

$F_T = 29.43 \text{ N} + 0.0688 \text{ N}$

$= \boxed{29.5 \text{ N}}$

↑ I have a feeling this may be wrong because the magnitude is awfully small...

It actually is!

3.5

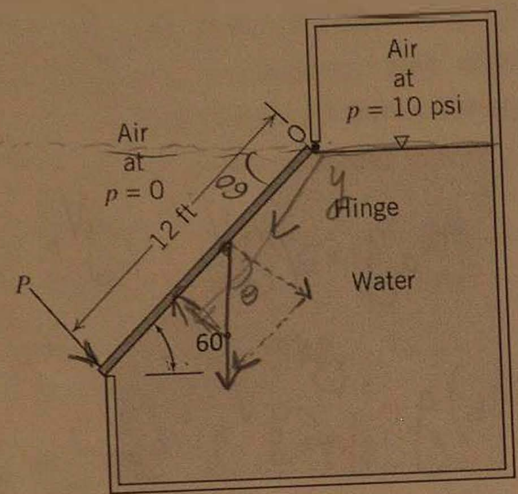
144

CHEE 314: Fluid Mechanics **Quiz #1 (2018)**

Q2. Calculate the minimum force P necessary to hold a uniform 12 ft square gate weighing 200 lb closed on a tank of water under a pressure of 10 psi. Drawing not to scale, state any assumptions. **[5 points]**

$w = 12\ ft$

200 lbf



$\Sigma M = 0$

Moment caused by weight

$$M_w = \underbrace{F_g \cos\theta}_{\text{vertical component}} \ \underbrace{\frac{L}{2}}_{\text{L acts @ center of mass}}$$

center of mass ~~centroid~~

$$M_w = (200\ lbf)(\cos 60)\left(\frac{12\ ft}{2}\right) = 600\ lbf \cdot ft$$

$$10\ psi = \frac{10\ lbf}{in^2} \cdot \frac{144\ in^2}{1\ ft^2} = 1440\ \frac{lbf}{ft^2}$$

air component

$$F = 1440\ \frac{lbf}{ft^2} \cdot (12\ ft)^2 = 2.0736 \times 10^5\ lbf$$

water component:

$$y_c = \frac{L}{2} = 6\ ft$$

$$F_R = \rho g \sin\theta\, y_c A$$

$$= (1.94\ slugs/ft^3)(32.2\ \tfrac{ft}{s^2})\sin(60)(1.76\ ft)(144\ ft^2)$$

$$= 4.674 \times 10^4\ lbf$$

→ total $F_{pressure} = F_R + F_{air} = 2.541 \times 10^5\ lbf$

acts @ y'

$$y' = y_c + \frac{I_{xx}}{y_c A}$$ X $P_0 \neq 0$.

$$= 6 + \frac{\frac{1}{12}(12)(12)^3}{6 \cdot 144} = 8\ ft$$

$\Sigma M = 0$

$$0 = F_p(12\ ft) + M_w - (8\ ft)(2.541 \times 10^5)$$

$$F_p = \frac{(8)(2.541 \times 10^5) - 600}{12}$$

$$\boxed{F_p = 1.693 \times 10^5\ lbf}$$

3

Right track

Q3. Reflection Statement (instructions to follow next week). **[1 point]**